Sustainability of a stage-structured exploited prey-predator system

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Abstract. In this paper, we investigate the effects of species exploitation to reach maximum sustainable yield (MSY) and maximum sustainable total yield (MSTY) in a stage structured prey-predator model. It is observed that harvesting only the prey or mature predator at MSY level never causes the extinction of the predator species. On the other hand harvesting of prey and mature predator under combined harvesting effort to reach MSTY (maximum sustainable total yield) may cause the extinction of either prey or predator. It is also shown that under independent harvesting effort, if MSTY does not exists, then it is due to the extinction of the predator species.

Key words: Harvesting, Stage structure, Maximum sustainable yield, Maximum sustainable total yield, Species extinction.

1 Introduction

The exploitation and management of biological resources like fisheries, forestry, and wildlife are popular research areas in recent years. In particular, fisher has satisfied the food needs of mankind for thousands of years and has become economically, socially and...
culturally fundamental resource. Increase demand of fish for world populations, destruction of fish habitats, growing of aquaculture fishery and most importantly the use of modern fish technology and environmental fluctuation cause the reduction in productivity of world fisher. Therefore there is a global concern to protect the ecosystem at large. In particular, the management of a fisher is a decision with multiple objectives like provision of the good bio-mass yield, conservation of fish population, good economic returns and recreation.

In recent years, the optimal management of renewable resources, which has a direct relationship to sustainable development, has been studied extensively by many authors Huimin (1998), Wang and Chen, (1997), Meng and Ke (1998), Meng and Ke (2001). But in the natural world, there are many species whose individual members have a life history that takes them through two stages, immature and mature. Previously, the stage-structured model of population growth consisting of immature and mature individuals was analyzed in the literature Zaghrout and Attalah (1996), Aiello et al. (1992), Murray (1989).

Though MSY policy is a simple way to manage single species population, but this policy faces a great challenge when it is introduced into a multispecies system. In fact, MSY policy may cause extinction in prey-predator systems (Ghosh and Kar, 2013), independent communities (Legovic and Gecek, 2010), mutualistic systems (Legovic and Gecek, 2012) and competitive systems (Gecek and Legovic, 2012). May et al. (1979) concluded that harvesting of both the prey and predator species under independent efforts ensures the extinction of predator species and they recommended to harvest the predator species for yield maximization. Matsuda and Abrams (2006) showed that MSY policy, in several food chain and food web models, causes extinction of top level predator and suggested that profit maximization would become more appropriate for sustainable fishing activity. Legovic et al. (2010) demonstrated that predator species will be driven to extinction if MSY is going to achieve from prey species only, but predator harvesting at MSY level is a sustainable harvesting in traditional prey-predator system. In addition, their results also reveal that harvesting of both the prey and predator at maximum sustainable total yield (MSTY) level under combined harvesting effort will cause the extinction of the predator species. Legovic and Gecek (2010) showed that harvesting of two independent population under independent effort never cause the extinction of either species. Recently, Kar and Ghosh (2013) showed that harvesting the prey species may or may not cause the extinction of the predator species if intra-specific competition among the predator is very strong. They also demonstrated that harvesting the prey species at MSY level never causes extinction of the predator species in ratio-dependent prey-predator system. More recently, Matsuda and Abrams (2013) mentioned that prey harvesting may cause oscillation in prey-predator system in the presence of weak intra-specific competition among the predator. They also showed that prey harvesting at MSY level will cause extinction of the predator species only. In such case feedback control would be more effective for conservation of both the prey and predator As the intra-specific competition enhances the mortality rate of the species, it is expected that the MSY policy will be significantly differ from the MSY policy in a traditional prey-predator system.

Stage-structured models have received much attention in recent years. Many consumer species go through two or more life stages as they proceed from birth to death, Kar (2005) considered a prey-predator model with stage-structure for predator and obtain conditions for permanence of the system. Recently, Chakraborty et al. (2011) proposed a prey-predator model with stage-structure for prey and harvesting on matured prey and predator. They observe the singularity induced bifurcation when variation of the economic interest of harvesting is taken into account. They also considered harvesting effort as a control parameter to formulate the optimal control problem under the dynamic framework towards optimal utilization of the resource. Jing and Ke (2004) measured the yield in single species model composed with two subpopulations: mature and immature. They measure the maximum yield from (i) mature population, (ii) immature population and (iii) both the mature and immature populations. We would like to address these issues in a prey-predator system with stage structure for predator.

In the present contribution we study the impact of MSY policy in a stage structured population model. Model description and related assumptions are given in Sec.2. Sec.3. is devoted to measure the impact of MSY policy from extinction viewpoint. In Sec.4, we investigate the role of MSTY policy under combined harvesting effort on both the prey and mature predator. Our outcomes are compared with the existing literature in sec.5.

2 The model

We now formulate a prey-predator system considering two stages for the predator species as follows
Maximum sustainable yield (MSY) = \frac{k}{\beta} \left(1 - \frac{x}{k}\right) - \alpha x, \\
\frac{dy}{dt} = \beta z - ry, \\
\frac{dz}{dt} = -rz + m\alpha xz + \gamma y - \delta z^2,

where \( x \) is the prey biomass, \( y \) is the biomass of the immature predator and \( z \) is the biomass of the mature predator population at time \( t \). \( k \) is the environmental carrying capacity and \( r_1 \) is the intrinsic growth rate of the prey species. \( \beta \) is the recruitment rate of immature predator. \( r_2 \) is the death rate of the immature predator. \( r_3 \) is the death rate of the mature predator, \( \gamma \) is the rate of conversion from immature predator to mature one, \( \delta \) is intra specific competition among the mature predator species. \( \alpha \) is the predation rate and \( m \) is the conversion coefficient.

The above ecological system has four steady states (for details see Kar, 2005). The only coexisting equilibrium \( P(x^*, y^*, z^*) \) where

\[ x^* = \frac{k(r_1 \beta + k r_2 a - ry)}{r_2 (km^2 + \delta r_1)}, \quad y^* = \frac{\beta r_1 (\beta r_2 k m a - r_1 r_2)}{r_2^2 (km^2 + \delta r_1)} \quad \text{and} \quad z^* = \frac{r_1 (\beta r_2 k m a - r_1 r_2)}{r_2 (km^2 + \delta r_1)} \]

exists provided \( \alpha \beta \gamma + kmr_2 \alpha^2 > r_2 r_3 \alpha > \alpha \beta \gamma - \delta r_1 r_2 \).

For stability analysis see Kar (2005). Now we shall discuss about the impacts of MSY and MSTY policy under different combinations of exploitation.

### 3 Impact of harvesting at MSY level

In this section we study the impacts of harvesting at MSY level either on prey species or mature predator species.

#### 3.1 Harvesting the prey population

If we harvest the prey species only taking harvesting function \( h = ex \) then the system (2.1) becomes:

\[ \frac{dx}{dt} = r_1 x \left(1 - \frac{x}{k}\right) - \alpha x - ex, \]
\[ \frac{dy}{dt} = \beta z - ry, \]
\[ \frac{dz}{dt} = -rz + m\alpha xz + \gamma y - \delta z^2. \] (3.1)

The only interior equilibrium, \( P(x^*, y^*, z^*) \) where

\[ x^* = \frac{k(r_1 \beta + k r_2 a - ry)}{r_2 (km^2 + \delta r_1)}, \quad y^* = \frac{\beta r_1 (\beta r_2 k m a - r_1 r_2)}{r_2^2 (km^2 + \delta r_1)} \quad \text{and} \quad z^* = \frac{r_1 (\beta r_2 k m a - r_1 r_2)}{r_2 (km^2 + \delta r_1)} \]

exists provided \( (r_1 r_3 \alpha + \delta r_1 r_2) > (\alpha \beta \gamma + \delta r_1 \beta) \) and \( r_1 (\beta r_2 k m a \alpha) > (r_1 r_2 r_3 + \delta k m r_2 \alpha). \)

The yield at equilibrium is \( Y = ex^* = \frac{(r_1 r_2 \alpha + k r_2 a - ry)}{2 r_2 (km^2 + \delta r_1)} \).

We see that the yield \( Y \) is a quadratic function of the harvesting effort \( e \) and it has a maximum at

\[ e_{\text{MSY}} = \frac{(r_1 r_2 \alpha + k r_2 a - ry)}{2 r_2 (km^2 + \delta r_1)}. \]

At \( e = e_{\text{MSY}} \) the population becomes,

\[ x_{\text{MSY}} = \frac{k(r_1 \beta + k r_2 a - ry)}{2 r_2 (km^2 + \delta r_1)}, \quad y_{\text{MSY}} = \frac{\beta r_1 (\beta r_2 k m a - r_1 r_2)}{2 r_2^2 (km^2 + \delta r_1)} \quad \text{and} \quad z_{\text{MSY}} = \frac{r_1 (\beta r_2 k m a - r_1 r_2)}{2 r_2 (km^2 + \delta r_1)}. \]

Maximum sustainable yield (MSY) = \( \frac{k(r_1 \beta + k r_2 a - ry)^2}{4 \beta r_1 (\beta r_2 k m a - r_1 r_2)} \).

When \( e = 0 \), the prey population at equilibrium

\[ x^* = \frac{k(r_1 \beta + k r_2 a - ry)}{r_2 (km^2 + \delta r_1)}. \]
and when $e = e_{\text{MSY}}$, the prey population

$$x_{\text{MSY}} = \frac{k(r_2 + \beta r_2 - \alpha \gamma \delta)}{\Delta_2}. $$

Hence it is observed that prey population reduces to its half of the value at MSY than unexploited system and predator population is
decreased by $\frac{\text{constant}(r_2 - \alpha \gamma \delta + r_1 \gamma)}{\Delta_2 \beta}. $ Therefore, we may state the following theorem.

**Theorem 1.** The system (3.1) is sustainable if the prey species is harvested at MSY level.

Now we illustrate this matter with an example, taking the ecological parameter as $r_1 = 2, r_2 = 0.4, r_3 = 0.1, k = 10, m = 0.5, \alpha = 0.5, \beta = 0.25, \gamma = 0.9, \delta = 0.5$ in appropriate units. Here we get $MSY = 1.31$ and it occurs for $e_{\text{MSY}} = 0.77$ with stable coexistence equilibrium at $(1.71, 1.11, 1.78)$ as shown in the Fig.1.

![Fig. 1. MSY exist when only prey population is harvested.](image)

### 3.2 Harvesting the mature predator population

If only the mature predator is subject to harvesting with effort $e$, then the system (2.1) becomes:
The coexisting equilibrium \( \hat{P}(\hat{x}, \hat{y}, \hat{z}) \) where

\[
\begin{align*}
\frac{dx}{dt} &= r_1 x \left(1 - \frac{x}{k}\right) - \alpha x z, \\
\frac{dy}{dt} &= \beta z - r_2 y, \\
\frac{dz}{dt} &= -r_3 z + m \alpha x z + \gamma y - \delta z^2 - \epsilon z,
\end{align*}
\]

exists provided \( 2r_2 r_3 \alpha + 5r_2 - 2 \alpha \beta - k \delta > 0. \)

The yield at equilibrium \( Y = e^2 = \frac{r_1 \beta r_3 (r_1 - r_2 - r_3 \gamma)}{r_2 (r_2 + b r_2)} \) which is a quadratic function of \( e \) and has a maximum at \( e = e_{\text{MSY}} \), where

\[
e_{\text{MSY}} = \frac{k \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)}.
\]

At \( e = e_{\text{MSY}} \) the population becomes,

\[
x_{\text{MSY}} = \frac{k \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)}, \quad y_{\text{MSY}} = \frac{\beta r_3 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)} \quad \text{and} \quad z_{\text{MSY}} = \frac{k \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)}.
\]

and \( \text{MSY} = \frac{r_1 \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)}. \)

We have seen earlier that when \( e = 0 \),

\[
x^* = \frac{k \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)}, \quad y^* = \frac{\beta r_3 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)} \quad \text{and} \quad z^* = \frac{r_1 \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)}.
\]

Hence it is observed that at \( e = e_{\text{MSY}} \) the prey population is increased and increased by \( \frac{r_1 \beta r_2 (r_2 - r_3 \gamma)}{2 r_2 (r_2 + b r_2)} \) and predator population is decreased and decrease to exactly half of its value.

Since all the species persist to reach the MSY, we can state the following.

**Theorem 2.** The system (3.2) is sustainable when the mature predator is harvested at its MSY level.

Now we shall discuss with an example taking the ecological parameter as \( r_1 = 1, r_2 = 0.3, r_3 = 0.1, k = 5, m = 0.2, \alpha = 0.1, \beta = 0.25, \gamma = 0.9, \delta = 0.3 \) in appropriate units. For the above set of parameters we get \( \text{MSY} = 0.45 \) and it occurs for \( e_{\text{MSY}} = 0.38 \) with stable coexistence equilibrium at (4.4.101, 1.21) as shown in the Fig. 2.
4 Combined harvesting of the prey and mature predator population

If we consider combined harvesting to the prey and mature predator based on catch per unit effort (CPUE) hypothesis, the system (2.1) becomes:

\[
\frac{dx}{dt} = r_1 x \left(1 - \frac{x}{k}\right) - \alpha xz - q_1 ex,
\]
\[
\frac{dy}{dt} = \beta z - r_2 y,
\]
\[
\frac{dz}{dt} = r_3 z + ma x z + \gamma y - \delta z^2 - q_3 ez.
\]

(4.1)

Here the coexisting equilibrium \( P(\hat{X}, \hat{Y}, \hat{Z}) \) where

\[
\hat{X} = \frac{k(\alpha_2 - C)}{r_2}, \quad \hat{Y} = \frac{\beta (E_1 - q_2 D)}{r_1 E} \quad \text{and} \quad \hat{Z} = \frac{E_1 - q_2 D}{r_2 E}
\]
assuming \( A = q_3 \alpha - q_1 \delta, B = km \alpha^2 + r_1 \delta, C = \alpha \beta \gamma - r_2 r_3 \alpha - r_1 r_2 \delta, D = q_1 r_1 + km q_1 \alpha, E = \beta \gamma + km r_3 \alpha - r_2 r_3, \)
exists provided \( Ar_2 - C > 0 \) and \( E r_1 - c r_3 D > 0. \)

The yield at equilibrium 
\( Y = e(q_1 \delta + q_2) = \frac{e^2Fr_2 - \epsilon(kqC - qr_1 e)}{Fr_1} \)
where \( F = k(1 - m)q_1 \alpha - q_1^2 r_1 - kq_1^2 \delta. \)
Since it is a quadratic function of \( e, \) it has a unique maximum at \( e = e_{MSY} \) and 
\( e_{MSY} = \frac{kq_1^2 e + q_1 r_1 e}{2kq_2}. \)

At \( e = e_{MSY}, \) the population becomes,
\[ x_{MSY} = \frac{4kq_1 C - q_1 r_1 e - 2kq_2}{2Fr_1}, \]
\[ y_{MSY} = \frac{2kq_1 C - q_1 r_1 e}{2Fr_1}, \]
and \( MSTY = \frac{F(kqC - qr_1 e) - (kqC - qr_1 e)^2}{2Fr_1}. \)

Our motivation is to reach at MSTY level in such a way that no species will go to extinction. Here it is observed that the prey exists at MSTY level.

if \( F > 0, A(kq_1 C - q_1 r_1 E) - 2CF > 0 \) or if \( F < 0, A(kq_1 C - q_1 r_1 E) - 2CF < 0 \)
and predator exists at MSTY level

if \( F > 0, 2r_1 EF - D(kq_1 C - q_1 r_1 E) > 0 \) or if \( F < 0, 2r_1 EF - D(kq_1 C - q_1 r_1 E) < 0 \).

So we can conclude that,

(a) The system (4.1) is a sustainable policy i.e., no species goes to extinction to reach MSTY if any one of the following two conditions hold

(i) \( F > 0, A(kq_1 C - q_1 r_1 E) - 2CF > 0 \) and \( 2r_1 EF - D(kq_1 C - q_1 r_1 E) > 0 \)
(ii) \( F < 0, A(kq_1 C - q_1 r_1 E) - 2CF < 0 \) and \( 2r_1 EF - D(kq_1 C - q_1 r_1 E) < 0. \)

(b) MSTY does not exist due to extinction of prey if \( F > 0, A(kq_1 C - q_1 r_1 E) - 2CF < 0 \) or if \( F < 0, A(kq_1 C - q_1 r_1 E) - 2CF > 0. \)
(c) MSTY does not exist due to extinction of predator if \( F > 0, 2r_1 EF - D(kq_1 C - q_1 r_1 E) < 0 \) or if \( F < 0, 2r_1 EF - D(kq_1 C - q_1 r_1 E) > 0. \)

Hence the following three cases may arise attempting to reach MSTY.

(i) MSTY exist, all the species persist in sustainable policy.
(ii) MSTY does not exist due to extinction of both the predator
(iii) MSTY does not exist due to extinction of prey species.

Now we analyze all these three cases by numerical simulations.

Case I. MSTY exists.

Taking the ecological parameter as \( r_1 = 1, r_2 = 0.3, r_3 = 0.1, k = 5, m = 0.2, \alpha = 0.1, \beta = 0.25, \gamma = 0.9, \delta = 0.3, q_1 = 0.7, q_3 = 0.9 \)
we get \( MSTY = 1.399 \) and it occurs at \( e_{MSY} = 0.58 \) with stable coexistence equilibrium at \((2.67, 0.51, 0.61)\) as shown in the Fig.3.
It is also to be noted that here \( F < 0, A(kq_1 C - q_1 r_1 E) - 2CF < 0 \) and \( 2r_1 EF - D(kq_1 C - q_1 r_1 E) < 0. \)
Fig. 3. Yield curve and biomass of the mature and immature predators. MSTY exists and as effort increases biomass of all the species decrease.

Case II. MSTY does not exist due to extinction of the predator

If we make only one change taking the ecological parameter \( r_2 = 0.8 \) instead of \( r_2 = 0.3 \) and all others parameters remains same, it is observed that harvesting the prey and mature predator species at MSTY level causes the extinction of both predators at \( e = 0.29 \) whereas maximum yield occurs at \( e = 0.48 \). (Fig. 4). It is also to be noted that, here \( F < 0 \), \( A(kq_1C - q_1r_1E) - 2CF < 0 \) and \( 2r_1EF - D(kq_1C - q_1r_1E) < 0 \).
Fig. 4. MSTY does not exist due to extinction of both the predators at $e = 0.8152$.

Case III. MSTY does not exist due to extinction of prey species.

Now it is illustrated by taking the ecological parameters as $r_1 = 0.25, r_2 = 0.3, r_3 = 0.1, k = 5, m = 0.2, \alpha = 0.1, \beta = 0.25, \gamma = 0.9, \delta = 0.3, q_1 = 0.7, q_3 = 0.9$. Here $r_1 = 0.25$ instead of $r_1 = 1$ and all others parameters remain same as in case I where MSTY exist and it is observed that prey species goes to extinction at $e = 0.083$ but both the predator may persist whereas maximum yield occurs at $e = 0.149$. It is seen that $F < 0, A(kq_1C - q_3r_1E) - 2EF > 0$ and $2r_1EF - D(kq_1C - q_3r_1E) < 0$ holds. So MSTY does not exist due to extinction of the prey population at MSTY level as shown in the Fig. 5.
Fig. 5. Yield curve and biomass of the prey and both the predator. Biomass of both prey and predators decrease as the effort increases and MSTY does not exist due to extinction of prey population.

5 Harvesting the prey and mature predator population with independent harvesting effort

In this section we assume the stage structured population model (2.1) in the presence of independent harvesting efforts as

\[
\frac{dx}{dt} = r_1 x (1 - \frac{x}{k}) - \alpha x z - q_1 e_1 x, \\
\frac{dy}{dt} = \beta z - r_2 y, \\
\frac{dz}{dt} = r_3 z + ma x z + g y - \delta z - q_2 e_3 z.
\]  

(5.1)

Here yield is maximized with respect to \( e_1 \) and \( e_3 \) and total maximum sustainable yield is obtained. Maximum sustainable total yield (MSTY) must satisfy \( \frac{\partial Y}{\partial e_1} = 0 \) and \( \frac{\partial Y}{\partial e_3} = 0 \) in addition Hessian Matrix to be negative definite.

The Hessian Matrix \( H \) corresponding to \( Y \) is
\[ H = \begin{pmatrix} \frac{\partial^2 Y}{\partial e_1^2} & \frac{\partial^2 Y}{\partial e_1 \partial e_3} & \frac{\partial^2 Y}{\partial e_3^2} \\ \frac{\partial^2 Y}{\partial e_1 \partial e_3} & \frac{\partial^2 Y}{\partial e_3^2} & \frac{\partial^2 Y}{\partial e_2^2} \end{pmatrix} \]

and it will be negative definite if and only if the principle minors viz.

\[ D_1 = \frac{\partial^2 Y}{\partial e_1^2} < 0 \text{ and } D_2 = \det(H) > 0. \]

In our problem

\[ D_1 = \frac{\partial^2 Y}{\partial e_1^2} = -\frac{2k^2 q_1^2}{(kma^2 + r_1 d^2)} \]

and

\[ D_2 = \det(H) = \frac{q_1^2 q_3^2 (4r_1 \delta - k\alpha^2 (1-m)^2)}{(kma^2 + r_1 d^2)^2}. \]

Hence \( D_1 < 0 \) and \( D_2 > 0 \) if \( 4r_1 \delta > k\alpha^2 (1-m)^2 \), which is the necessary condition for the existence of global MSTY.

To analyze the situation numerically we take \( r_1 = 1, r_2 = 0.3, r_3 = 0.1, k = 5, m = 0.2, \alpha = 0.1, \beta = 0.25, \gamma = 0.9, \delta = 0.3, q_1 = 0.1 \) and \( q_3 = 0.9 \). It is seen that \( 4r_1 \delta > k\alpha^2 (1-m)^2 \) occurs and we see that neither prey nor any predator population becomes extinct at MSTY level. MSTY is 1.42 with no species extinction and coexisting equilibrium is \((2.34, 0.64, 0.77)\). It can be concluded from the following table that MSTY is independent with respect to cathability coefficients. So global MSTY exists.

<table>
<thead>
<tr>
<th>( q_1 )</th>
<th>( q_3 )</th>
<th>( e_1 )</th>
<th>( e_3 )</th>
<th>Prey Immature Predator</th>
<th>Mature Predator</th>
<th>MSTY</th>
</tr>
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<tbody>
<tr>
<td>0.7</td>
<td>0.8</td>
<td>0.65</td>
<td>0.58</td>
<td>2.34</td>
<td>0.64</td>
<td>0.77</td>
</tr>
<tr>
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<td>0.9</td>
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<td>0.52</td>
<td>2.34</td>
<td>0.64</td>
<td>0.77</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>0.91</td>
<td>0.66</td>
<td>2.34</td>
<td>0.64</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Fig. 6. Yield surface with respect to the independent harvesting efforts \( e_1 \) and \( e_3 \). Global MSTY exists at \( e_1 = 4.53 \) and \( e_3 = 0.52 \).
Fig. 7. Variation of prey biomass with respect to $e_1$ and $e_3$.

Fig. 8. Immature predator biomass as a function of $e_1$ and $e_3$. 
In another situation with $r_1 = 1, r_2 = 0.8, r_3 = 0.1, k = 5, m = 0.2, \alpha = 0.1, \beta = 0.25, \gamma = 0.9, \delta = 0.3, q_1 = 0.5, q_3 = 0.7$, it is seen that $4r_1\delta > km \alpha^2 (1 - m)^2$ occurs but MSTY does not exist due to extinction of predators species. In this case maximum yield (MY) occurs at $e_1 = 1.00385$ and $e_3 = 0.3443$.

Hence we can conclude that $4r_1\delta > km \alpha^2 (1 - m)^2$ is the necessary but not sufficient condition for the existence of global MSTY.
Fig. 10(a). Represents yield surface with respect to $e_1$ and $e_3$ in the region $R = \{(e_1, e_3) : e_1 \in (0, 1.003) \text{ and } e_3 \in (0, 0.33)\}$ till the predator species goes to extinction and (b) represents yield after extinction of predator species.

Fig. 11. Variation of prey biomass with respect to $e_1$ and $e_3$. 
6 Conclusion

This article describes the impacts of harvesting to reach the MSY or MSTY level in a stage structured population model. Some of the results are derived analytically and some others are shown numerically using the simulated parameter sets. It is observed that if prey is the only target species for harvesting, then MSY always exists. In this situation prey biomass reduces to exactly half of its value.
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in compared to unexploited system. Legovic et al. (2010) showed that in traditional prey predator systems, fishing to reach MSY of the prey population only will cause the extinction of predator population but in this case it is a sustainable policy (see Fig.1). It is also shown that harvesting the mature predator to reach MSY is a sustainable policy but predator biomass reduces exactly half of the unexploited system. One of our important investigation is to reach MSTY when combined harvesting effort is imposed to the prey and the mature predator. We have analyzed the impacts of common harvesting effort analytically and also illustrated numerically. It is observed that MSTY may exist (see Fig.3), and if it dose not exist then it is due to extinction of either the predator (see Fig.4), or prey species (see Fig.5).

Lastly, we have discussed about the global MSTY taking independent harvesting efforts. It is seen that if global MSTY exists, it is independent of catchability coefficients. This result is a great achievement for a stage structured population model. Fig. 3 shows that MSTY under combined harvesting effort exists and is equal to 1.399 where as the same system exhibits the result that the global MSTY if exists is equal to 1.42 which is larger than MSTY under combined harvesting effort. In another simulation it is proved that though Hessian Matrix with respect to the selective harvesting effort is negative definite, i.e., necessary condition for global MSTY is satisfied but due to extinction of predators species global MSTY does not occur for harvesting the prey and mature predator.

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