# Fundamentals of Nanofluids: Evolution, Applications and New Theory

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**Abstract.** *In the past decade, quick developments in nanotechnology have created quite a lot of prospect for the scientists and engineers to check up. Nanofluid is one of the amazing consequences of such progression. Nanofluids are engineered by suspending nanoparticles with average sizes below 100 nm in traditional heat transfer fluids such as water, oil, and ethylene glycol etc. Nanofluids are considered to offer important advantages over conventional heat transfer fluids. A very small amount of guest nanoparticles, when dispersed uniformly and suspended stably in base fluids, can provide dramatic improvements in the thermal properties of host fluids. Stable and highly conductive nanofluids are produced by generally, one-step and two-step production methods. Both approaches to creating nanoparticle suspensions suffer from agglomeration of nanoparticles, which is a key issue in all technology involving nanopowders. In trying to understand the unexpected discoveries and so to overcome the limitations of classical models, a number of investigators have proposed new physical concepts, mechanisms, and developed new models for enhancing the transport properties. In the present work, wide-ranging fundamental evolution of nanofluids have been discoursed thoroughly by sketching out a gargantuan depiction of the diminutive biosphere of nanofluids through a brief review of some chronological foremost milestone such as the concepts of nanofluids, the preparations and performances of nanofluids, conductivity, viscosity and density correlations of nanofluids, and potential applications and benefits of nanofluids. Also, different kinds of modeling and very important slip mechanisms of constructing heat transfer modeling of nanofluids have been discussed comprehensively in this study. Furthermore, we have established*

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*new mathematical equations theoretically for electrical conductivity and thermophoretic velocity in nanofluids as well as nanoparticles mass flux equation due to Brownian diffusion and thermal diffusion which can be used for general transport nanofluids modeling. Our results of thermal diffusion coefficient are also justified by the experimental findings.*

**Key words:** Nanofluid, conductivity, viscosity, diffusion, thermophoresis, nanofluid applications

## 1 Introduction on Nanofluids

Nanofluid is a fluid mixing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. Nanoparticles which are commonly used in nanofluids are made from numerous materials such as oxide ceramics ($Al_2O_3, CuO$), nitride ceramics ($AlN, SiN$), carbide ceramics ($SiC, TiC$), metals ($Cu, Ag, Au$), semiconductors ($TiO_2, SiC$), carbon nanotubes; and composite materials such as alloyed nanoparticles, $Al_{70}Cu_{30}$ or nanoparticle core-polymer shell composites. In addition, nonmetallic, metallic, and other materials for nanoparticles, completely new materials and structures may also have desirable characteristics. Common base fluids include water ($H_2O$), ethylene glycol ($EG$), engine oil ($EO$), pump oil and glycerol have been used as host liquids in nanofluids. Enhancing thermal conductivity of liquids, concepts of nanofluids, modeling on suspensions are the most recent approach in more than a century of work. The low thermal conductivity of conventional heat transfer fluids (HTFs) is a serious constraint in improving the performance and compactness of engineering equipment. The notion of particles which can convert from millimeter to micrometer sized in the late $19^{th}$ century has become a part of the modern adventure into the new realm of the very small: the world of nanoparticles. Particularly, nanofluids are a new noble class of nanotechnology based heat transfer fluids that are engineered by stably suspending a small amount, 1 volume % or less of particles with lengths on the order of in traditional heat transfer fluids. In the commencement of 1990s, Choi had anticipated the impression of the emergence of novel class of fluid and the term 'nanofluid' (Choi [1]). The working together of two things to produce an effect greater than the sum of their individual effects is the key to success of nanofluids. The result is a matching set of remarkable thermal properties that have motivated students, scientists, and engineers around the world to undertake the challenge of exploring the possibilities of these captivating fluids.

## 2 Brief History of Nanofluids

To improve the heat transport properties of fluids, enormous activities are being carried out. It is well known that many solid metallic and non-metallic materials have much higher thermal conductivities than normal HTFs. Thus, it is an innovative idea, trying to enhance the thermal conductivity by adding solid particles into HTFs since Maxwell initiated it before (Maxwell [2]). At the very beginning, more initiatives have been taken to produce mixtures. Solid particles of micrometer, even millimeter magnitudes were blended into the base fluids to make suspensions or slurries. However, large solid particles cause troublesome problems, such as abrasion of the surface, clogging the microchannels, eroding the pipeline and swelling the pressure drop, which substantially limits the practical applications. Actually, liquid suspension was primarily a theoretical treatment only of some hypothetical interest, and subsequent studies by other researchers achieved inconsequential success. The large size of the particles and the difficulty in production of small particles were the most notable limiting factors. The situation became fruitful when Choi and Eastman in Argonne National Laboratory revisited this field with their innovative idea of nanoscale metallic particle and carbon nanotube suspensions. They have experimented with various metal and metal oxides nanoparticles in different base fluids and the results were promising, however, many things remained elusive about these suspensions of nano-structured materials.

The egression of nanofluids as a new field of nanoscale heat transfer in liquids is directly related to miniaturization trends and nanotechnology. A short history of Argonne National Laboratory (ANL) is described to show that the program has embraced a wide range (meters to nanometers) of size regimes and how a wide range of research has become narrow, starting with large scale and coming down through microscale to nanoscale in this program, climaxing in the invention of nanofluids. Table-1 summarizes a brief history of the emergence of nanofluids as an infant field of inquiry.

Table-1: Chronology of emersion of nanofluids

| | |
|---|---|
| 1981 | D. B. Tuckerman familiarizes microchannel technology. |
| 1985 | Argonne National Laboratory initiated a scheme to establish advanced fluids. |
| 1991 | In Argonne National Laboratory, Choi progresses a microchannel heat exchanger for the settled photon source. |
| 1992 | Argonne national fluid program starts to work from micro to nano with funding cut. |
| 1993 | Thus, research of nanofluids technology came into being 12 years after Tuckermans introduction of microchannel technology. In May, Choi submits first nanofluids proposal and second proposal with Eastman later on. |
| 1994 | Third proposal of nanofluids submits. |
| 1995 | Choi presents a seminal paper on the concept of nanofluids at the American Society Mechanical Engineers Winter Annual Meeting, San Francisco, CA, Nov. 12-17. |
| 1998-Present | Argonne nanofluids research funded by U. S. Department of Energy (DOE) Office of Basic Energy Science (BES) and Office of Transportation Technologies to works on both fundamentals and applications of nanofluids. |
| 1999 | Lee's group publishes the first SCI article on nanofluids (Lee [3]). |
| 2000 | President William Jefferson "Bill" Clinton announces the United States' National Nanotechnology Initiative (NNI) at Caltech in January. Choi serves as a proposal reviewer for DOE BES and National Science Foundation. Also, he works on the first Nanoscale Science, Engineering and Technology proposals under the NNI. |
| 2001 | Choi's group with Eastman publishes two papers in Applied Physics Letters (Eastman et al. [4], Choi et al. [5] ). |
| 2002-2003 | The nanofluids research achieved a top position by DOE's Office of Basic Energy Sciences in both 2002 and 2003. |
| 2007 | The first single-theme conference on nanofluids was held. Nanofluids: Fundamentals and Applications, Copper Mountain, CO, Sept.16-20, 2007, the Engineering Conferences International. The first book on nanofluids, Nanofluids: Science and Technology (Das et al. [6]) is published by Wiley. |
| 2008-till now | Scientists all over the world are attempting to reach at an agreement by publishing their theoretical and experimental investigations so that the use of nanofluids will be effective as it was predicted before. |

# 3 An Archaic Concept with Unique Characteristics

In fact, nanofluids concept are centuries old. It can be found from the history that medieval artisans used a suspension of gold nanoparticles for their windows to make deep red color without knowing their own secrets. In the 15th century in Italy, ceramicist used metallic nanoparticles dispersed in a liquid to make shininess pottery (Padovani et al. [7]). These artisans were not aware of the size of the particles but they used them for color and luster. Though there are some significant differences between the usual colloids and nanofluids, nanofluids are a type of colloidal dispersion. Though someone can say that nanofluids are a type of colloidal dispersion but the particle size, phase, major physical properties, basic processing steps, and major applications are distinct from colloidal dispersions. Colloidal dispersion fluids have distinct properties such as (a) it contains 10-100 nm sizes particles, (b) a colloid in colloidal dispersions can be a solid, a liquid droplet, or a gas bubble, forming a suspension, an emulsion, or foam, respectively, (c) colloids have been studied primarily with a view to their rheological properties, (d) colloidal processing has five basic steps: powder synthesis, suspension preparation, consolidation, solvent removal, and densification at elevated temperatures and (e) colloid science has led to technological advances in ceramics. On the other hand, the properties of nanofluids which are different from colloidal fluids are as : (a) it has 1-100 nm sizes particles, (b) a nanoparticle in nanofluids is a solid, forming only a suspension, (c) the focus of the recent emergence of nanofluids has shifted to thermal properties, (d) nanofluids processing has one or two steps: powder synthesis and suspension preparation, and (e) nanofluids have recently emerged with new potential applications in cooling electronics, vehicle engines, nuclear reactors, energy efficiency enhancement, and biological sciences.

# 4 The Size of the Particles Matters

Particle size is very important in creating practical nanofluids before jumping into the specifics of the thermal properties of nanotechnology-based heat transfer fluids. There are some reasons to fix up the size of the particles in base fluids. First of all, particle size matters in making nanofluids stable. Dense nanoparticles can be suspended in liquids because the particles have an extremely high ratio of surface area to volume so that the interaction of the particle surface with the liquids is strong enough to overcome differences in density. Furthermore, nanoparticles are charged and thus particle-particle interactions are not allowed. Second, size matters in making nanofluids with novel properties. The very small particle size can affect transport mechanisms at the nanoscale. The properties of nanofluids are dominated not only by the characteristics of nanoscale surface and interface structures but also by nanoscale dynamics. Finally, particle size matters in making nanofluids useful. For example, since the size of nanoparticles is similar to that of bio-molecules, nanofluids can be used in biomedical applications such as drug delivery and nanofluids based control of biological functions.

# 5 Making Nanofluids

Dispersing the nanoparticles uniformly and suspending them stably in the host liquid is critical in producing high-quality nanofluids. Good dispersion and stable suspension are prerequisites for the study of nanofluids properties and for applications. The key in producing extremely stable nanofluids is to disperse monosized nanoparticles before they agglomerate. Many one-step and two-step physical and chemical processes have been developed for making nanofluids. Scientists are also constructing different process of making nanofluids. These processes can be summarized as follows:

## 5.1 One-Step Method

The one-step method is the process of combining the preparation of nanoparticles with synthesis of nanofluids, for which nanoparticles are directly prepared by physical vapor deposition (PVD) technique or liquid chemical method. In this method, the processes of desiccating storage, transportation, and discretely distribution of nanoparticles are avoided, so that the accumulation of nanoparticles is reduced and the stability of fluids is amplified. Synthesis and dispersion of nanoparticles into the base fluid take place simultaneously in one-step process. For example, Argonne developed one-step nanofluids production system in which nanoscale vapor from metallic source material can be directly dispersed into low vapor pressure fluids (Eastman et al. [4]). This novel one-step process was developed to overcome the van der Waals forces between the nanoparticles and produced stable suspensions of Cu nanoparticles without any dispersants. Another one-step physical process is wet grinding technology with bead mills. One-step physical method can't synthesize nanofluids in large scale, and the cost is also high, so the one-step chemical method is developing rapidly. Zhu et al. [8], Liu et al. [9] used one-step chemical reduction methods for producing stable metallic nanofluids. Also, Phuoc et al. [10] produced Ag - water nanofluid using one-step optical laser ablation in liquid. The main advantage of one-step process is that the nanoparticles are comparatively pure and have more uniform sizes. There are also some disadvantages for one- step method. The most important is that the residual reactants are left in the nanofluids due to incomplete reaction or stabilization. Also, only low vapor pressured base fluids are compatible with this process.

## 5.2 Two-Step Method

In two-step method, nanoparticles are first produced as a dry powder by physical or chemical methods such as inert gas condensation and chemical vapor deposition. Since 1930s, the inert gas evaporation-condensation technique (nanoparticles are formed via the evaporation of a metallic source in an inert gas) has been widely used in the synthesis of ultrafine metal particles. Also, the basic chemical method to make nanoparticles is to have some compound, typically a halide, containing a metal atom, as well as a reducing agent which removes the other parts of the compound. This step is followed by powder dispersion in the liquid. The main advantage of the two-step process is that nanoparticles can be produced separately by an industrial method that allows their massive and inexpensive production. This process is fairly well for oxide nanoparticles rather than metallic nanoparticles. The major problem of the two-step process is aggregation of nanoparticles. Kwak and Kim [11] showed that particles strongly aggregated before dispersion and also they aggregated after dispersion in ethylene glycol with 9 hours of sonication. Most researchers purchase nanoparticles in powder form and mix them with the base fluid. However, these nanofluids are not stable, although stability can be enhanced with controlling pH and adding surfactant. Some researchers purchase commercially available nanofluids. But these nanofluids contain impurities and nanoparticles whose size is different from vendor specifications.

## 5.3 Some Other Methods

Yu et al. [12] developed a continuous flow microfluidic micro-reactor to synthesize copper nanofluids. By this method, copper nanofluids can be continuously synthesized, and their microstructure properties can be varied by adjusting parameters such as reactant concentration, flow rate, and additives. Zhu et al. [13] showed that $CuO$ nanofluids with high solid volume fraction (up to 10 vol%) can be synthesized through a novel precursor transformation method with the help of ultrasonic and microwave irradiation. Chen and Wang [14] illustrated that phase transfer method is also a facile way to obtain monodisperse noble metal colloids.

## 6 Thermal Transport Properties

The pioneering experiments with nanofluids showed that nanofluids have new thermal transport phenomena that display remarkable enhancement in thermal properties such as thermal conductivity, thermal diffusivity, viscosity, convective heat transfer coefficients compared to those of base fluids. There are still searching a true correlation of thermal conductivity, viscosity and density though they have been extensively studied as has two phase and single phase heat transfer. Mathematical description of nanofluid properties are one of the enthusiastic phenomena. Among the many findings, some abnormal nanofluid behavior has been observed.

Firstly, unusual single phase convective heat transfer coefficient of nanofluids increases relative to the base fluid. Pak and Cho [15] reported heat transfer data in circular tubes for turbulent flow of alumina-water and titania-water nanofluids. Their results showed that Nusselt numbers up to about 29% higher than that of predicted by the pure fluid relation, even though the measured nanofluid properties were applied in defining the dimensionless groups in the correlation.

Secondly, anomalous viscosity of nanofluids enhances relative to the base fluids. Pak and Cho [15] calculated the viscosity of alumina-water and titania-water nanofluids and he observed that at 1-10 vol % of nanoparticles in pure water, viscosity is much higher than that of traditional water, well beyond the prediction of traditional viscosity models such as the model of Brinkman [16] or Batchelor [17]. The same observational conclusion has found in Maiga et al. [18].

Thirdly, comparative to the base fluids, atypical thermal conductivity of nanofluids increases significantly. Eastman et al. [4] reported that the thermal conductivity of ethylene-glycol increased 40% with 0.3 vol % copper nanoparticles of 10 nm diameters. Das et al. [19] observed 10-25% increases in water with 1-4 vol % alumina nanoparticles. Also, it has appeared that thermal conductivity of nanofluids is a powerful increasing function of temperature; much further so than that of pure liquids.

Even with several attempts, an agreeable explanation for the abnormal increase of the viscosity, thermal conductivity and uncharacteristic behavior of heat transfer coefficients in nanofluids is yet to be found. Pak and Cho [15], Xuan and Roetzel [20], and Xuan and Li [21] assumed that convective heat transfer enhancement is due mainly to dispersion of the suspended nanoparticles. However, Buongiorno [23] described particle dispersion and suggested that this effect is very small in nanofluids and thus cannot be explained the observed heat transfer enhancement. Xuan and Li [21] proposed that the enhancement could also come from intensification of turbulence due to the presence of the nanoparticles. However, Xuan and Li [21] and Pak and Cho [15] measured pressure drop and clearly observed that if the measured nanofluid viscosity is used, the turbulent friction factors in nanofluids can be well forecasted by the traditional friction factor correlations for pure fluids.

In short, nanofluids are a new field of scientific research that has grown enormously in the past few years. Still, it is a field in its adolescence, and there are a number of issues which have not been fully investigated. Despite recent advances, such as invention of unexpected thermal properties, proposed new mechanisms and unconventional model, the mysteries of nanofluids are unsolved. Nanofluids are an interdisciplinary ensemble of several fields of science and technology. Huge amount of work is necessary in every area of nanofluids from very basics to formulation to big-scale production.

## 6.1 Thermal Conductivity of Nanofluids

The theoretical investigation of the transport properties of heterogeneous mixtures have been of great interest since virtually the time of Maxwell. The reasons of this interest are, of course, the enormous variety of physical systems. Among various transport properties, magnetic permeability, comprising the dielectric constant, electrical conductivity, and thermal conductivity may be treated

together because of the nearly identical form of the conduction laws that control them. It is impossible to cover all the theoretical and experimental overviews of effective thermal conductivities of mixtures because these studies have been started since Maxwell's theoretical work published more than a century ago. It is evident from the many experimental and theoretical study that the thermal conductivity of nanofluids depends on the thermal conductivity of both base fluids and nanoparticles, the volume fraction, the shape and Brownian motion of the nanoparticles, the surface area, the temperature and the interfacial layer of solid-liquid suspensions. In the past decade, many efforts have been made to predict the enhancement of thermal conductivity based on experiments and theoretical analysis. An empirical equation called the mixture rule is generally employed by Nielsen [24] , Nan [25] as given by

$$\kappa_{eff}^n = (1-\phi)\kappa_{bf}^n + \phi\kappa_p^n \qquad (6.1)$$

which can be rewritten an alternative form

$$\kappa_{eff} = \left\{ 1 + \phi\left[(\kappa_p/\kappa_{bf})^n - 1\right]\right\}^{1/n}\kappa_{bf}, \quad -1 \le n \le 1 \qquad (6.2)$$

where, $\kappa_p$, $\kappa_{bf}$ represent the thermal conductivity of particles and bulk fluid respectively, $\phi$ is the particles volume fraction of the suspension and $\kappa_{eff}$ is the effective thermal conductivity of the mixture. For $n = 0$, the empirical equation becomes the parallel mixture rule (Wiener [26]) and for $n = -1$, the empirical equation becomes the series mixture rule (Wiener [26]). Landau and Lifshitz [27] considered a mixture as being a homogeneous and isotropic medium and obtained the equation for $n = 1/3$ by taking an average over a large volume:

$$\kappa_{eff} = \left\{ 1 + \phi\left[(\kappa_p/\kappa_{bf})^{1/3} - 1\right]\right\}^3 \kappa_{bf} \qquad (6.3)$$

Again for $n \to 0$, the equation (6.1) define as the logarithmic mixture rule (Lichtenecker [28]) as

$$\ln\kappa_{eff} = (1-\phi)\ln\kappa_{bf} + \phi\ln\kappa_p \qquad (6.4)$$

Maxwell was one of the first persons to investigate conduction analytically through suspension particles. Maxwell [2] considered a very dilute suspension of spherical particles by ignoring the interactions among the particles. He defined the effective thermal conductivity, $\kappa_{eff}$ as:

$$\kappa_{eff} = \kappa_{bf} + 3\phi\frac{\kappa_p - \kappa_{bf}}{2\kappa_{bf} + \kappa_p - \phi(\kappa_p - \kappa_{bf})}\kappa_{bf} = \frac{2\kappa_{bf} + \kappa_p - 2\phi(\kappa_{bf} - \kappa_p)}{2\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)}\kappa_{bf} \qquad (6.5)$$

which for low particle-volume concentrations can be further approximated as

$$\kappa_{eff} = \kappa_{bf} + 3\phi\frac{\kappa_p - \kappa_{bf}}{2\kappa_{bf} + \kappa_p}\kappa_{bf} \qquad (6.6)$$

Even though given this form, it should be noted that Maxwells equation is only first order approximation and applies only to mixtures with low particle-volume concentrations. Maxwells model shows that the effective thermal conductivity relies on only spherical particle, the bulk fluid and the volume fraction of the solid particles. Yu and Choi [29] proposed a modified Maxwell model considering the effect of nano-layer by replacing the thermal conductivity of solid particles $\kappa_p$ in equation (6.5) with the modified thermal conductivity $\kappa_{pe}$, which is based on the so called effective medium theory (effective medium approximations are descriptions of a medium (composite material) based on the properties and the relative fractions of its components and are derived from calculations):

$$\kappa_{pe} = \frac{2(1-\gamma) + (1+\beta)^3(1+2\gamma)\gamma}{\gamma - 1 + (1+\beta)^3(1+2\gamma)}\kappa_p \qquad (6.7)$$

where, $\gamma = \kappa_{layer}/\kappa_p$ is the ratio of nano-layer thermal conductivity to particle thermal conductivity and $\beta = h/r$ is the ratio of the nano-layer thickness to the original particle radius. Thus Maxwell equation (6.5) can be modified into the following form:

$$\kappa_{eff} = \frac{2\kappa_{bf} + \kappa_{pe} + 2\phi(\kappa_{pe} - \kappa_{bf})(1+\beta)^3}{2\kappa_{bf} + \kappa_{pe} - \phi(\kappa_{pe} - \kappa_{bf})(1+\beta)^3}\kappa_{bf} \qquad (6.8)$$

This modified correlation of thermal conductivity can assume the existence of the very thin nano-layer. It can also be concluded that the smaller the particles size, the higher the thermal conductivity. Moreover, the very small size nanoparticles should remarkably improve the stability of the mixer. Hamilton and Crosser [30] extended the Maxwell study of spherical particles and introduced shape factor which can be determined experimentally for different types of materials to cover the non-spherical particles. Hamilton and Crosser's thermal conductivity of heterogeneous two component systems is

$$\kappa_{eff} = \kappa_{bf} \frac{\kappa_p + (n-1)\kappa_{bf} - (n-1)\phi(\kappa_{bf} - \kappa_p)}{\kappa_p + (n-1)\kappa_{bf} + \phi(\kappa_{bf} - \kappa_p)} \tag{6.9}$$

where the empirical shape factor, $n$ is defined by $n = 3/\psi$ and $\psi$ is the sphericity defined as the ratio of the surface areas of a sphere with the volume equal to that of the particle. The Hamilton- Crosser model reduces to Maxwell model when $\psi = 1$. This model is valid as long as the conductivity of the particles is larger than the conductivity of the continuous phase at least by a factor of 100.

Xue [31] defined a model for the effective thermal conductivity of nanofluids considering the effect of the interfacial layer between solid and the base fluid based on the Maxwell model and average polarization theory as

$$9\left(1 - \frac{\phi}{\lambda}\right) \frac{\kappa_{eff} - \kappa_{bf}}{2\kappa_{eff} + \kappa_{bf}} + \frac{\phi}{\lambda}\left[\frac{\kappa_{eff} - \kappa_{c,x}}{\kappa_{eff} + B_{2,x}(\kappa_{c,x} - \kappa_{eff})} + \frac{4(\kappa_{eff} - \kappa_{c,y})}{2\kappa_{eff} + (1 - B_{2,x})(\kappa_{c,y} - \kappa_{eff})}\right] = 0 \tag{6.10}$$

where, $\lambda = abc/((a+t)(b+t)(c+t))$ and $a$, $b$, $c$ is half-radii of the assumed elliptical nanoparticles and $t$ is the thickness of elliptical shell, $\kappa_{c,x}$, $\kappa_{c,y}$ are the effective dielectric constant and $B_{2,x}$ is the depolarization factor along the $x$ symmetrical axis which has been derived from the average polarization theory. Xue [31] claimed that for nanotube in oil nanofluids at high thermal conductivity and nonlinearity with nanotube loadings, his model are in good agreements with the experimental data. However, Yu and Choi [32] found that the predicted values using the parameters are inaccurate. Xuan et al. [22] considered the random motion of nanoparticles with the Maxwell model and proposed a modified formula for the effective thermal conductivity as follows:

$$\kappa_{eff} = \frac{2\kappa_{bf} + \kappa_p - 2\phi(\kappa_{bf} - \kappa_p)}{2\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)}\kappa_{bf} + \frac{\rho_p c_p \phi}{2}\sqrt{\frac{k_B T}{3\pi\mu r_c}} \tag{6.11}$$

where, $k_B$ is the Boltzmann constant, $r_c$ is the apparent radius of clusters and depends on the fractal dimension of the cluster structure. Kumar et al. [33] considered large enhancement of thermal conductivity with strong temperature dependence Brownian motion and proposed a relation as

$$\kappa_{eff} = \kappa_{bf}\left[1 + c\bar{u}_p\frac{\phi r_m}{\kappa_{bf}(1 - \phi)r_p}\right] \tag{6.12}$$

where, $r_m$, $r_p$ are the radius of liquid molecule and particle respectively and they defined the thermal conductivity of the particles as $\kappa_p = c.\bar{u}_p$, $c$ is constant and it was proposed that the velocity of the particles can be taken due to Brownian motion at a given temperature of the suspension. Mean velocity of the nanoparticles has been calculated using the Stokes-Einstein's formula, the average particle velocity, $\bar{u}_p = 2k_B T/(\pi\nu d_p^2)$. However, the validity of the model in the molecular size regime has to be explored and it may not be suitable for a large concentration of the particles where interactions become an important matter. Jang and Choi [34] devised a theoretical model for predicting thermal conductivity of nanofluids. They considered the four modes of energy transport in nanofluids such as interactions among base fluid molecules ($\kappa_{bf}(1 - \phi)$), thermal diffusion of nanoparticles in base fluids ($\kappa_p \phi$), collision among nanoparticles due to Brownian motion (ignored) and the thermal interactions between nanoparticles and molecules of base fluids ($fh\delta_T$). Here, $h \approx (\kappa_{bf}/d_p)Re_{d_p}^2 Pr^2$ is the heat transfer coefficient, $\delta_T \approx \delta/Pr$ is the thermal boundary layer thickness, $\delta \approx 3d_{bf}$ and $d_{bf}$, $d_p$ are the diameter of the base fluid molecules and nanoparticles respectively. They defined the thermal conductivity of nanofluids as follows

$$\kappa_{eff} = \kappa_{bf}(1 - \phi) + \kappa_p\phi + 3C\frac{d_{bf}}{d_p}\kappa_{bf}\phi Re_{d_p}^2 Pr \tag{6.13}$$

Prasher et al. [35] written heat transfer coefficient, $h = \kappa_{bf}/a(1 + ARe^m Pr^{0.333}\phi)$ and the Reynolds number, $Re = \nu^{-1}\sqrt{(18\kappa_{bf}T)/(\pi\rho_p d_p)}$. Here, $a$ represent the radius of the nanoparticles, $A$ and $m$ are constants. Also, Prasher et al. [36] considered convection caused by Brownian motion of nanoparticles is primary responsible for the enhancement in the effective thermal conductivity of nanofluids. They proposed the modified Maxwell model as follows:

$$\kappa_{eff} = (1 + ARe^m Pr^{0.333}\phi)\kappa_{bf}\frac{2\kappa_{bf} + \kappa_p - 2\phi(\kappa_{bf} - \kappa_p)}{2\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)} \tag{6.14}$$

Koo and Kleinstreuer [37] proposed a correlation of thermal conductivity which has been taken into account the effect of particle size, particle volume fraction and temperature dependence with Brownian motion as

$$\kappa_{eff} = \kappa_{bf}\frac{2\kappa_{bf} + \kappa_p - 2\phi(\kappa_{bf} - \kappa_p)}{2\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)} + 5 \times 10^4\beta\phi\rho_p c_p\sqrt{\frac{k_B T}{\rho_p d_p}}f(T, \phi) \tag{6.15}$$

where, $\beta$ is related to particle motion, $f(T, \phi)$ is assumed to vary continuously with the particle volume fraction.

## 6.2 Viscosity of Nanofluids

In fact, the viscosity is one of the key properties of nanofluids. It is thought that viscosity is as critical as thermal conductivity in engineering systems because the nanofluid was probable to show an increase in thermal conductivity without an increase in pressure drop, which in turn is related to fluid viscosity. Viscosity of nanofluids has been investigated by many investigators concerning the factors influencing the nanofluids. These factors include the nanoparticle volume fraction, concentration, size of nanoparticles, temperature of nanofluids and share rate etc. The model for predicting the viscosity of nanofluids is also an important issue interested by many investigators. Using the standard Brooksfield viscometer, Pak and Cho [15] experimented with $\gamma Al_2O_3$ and $TiO_3$ particles of 13 and 27 nm average size suspended in water and found that the suspensions are Newtonian at very low particle volume fractions and start showing shear thinning behavior (i.e. decrease in viscosity with shear rate) with an increase in particle volume fraction. A large number of studies of viscosity are available on the theoretical modeling of suspension. The viscosity of suspension can be described as relative to the viscosity of the liquid phase:

$$\mu_{nf} = \mu_{bf} \mu_r \tag{6.16}$$

where, $\mu_{nf}$ and $\mu_{bf}$ are respectively the dynamic viscosity of the suspension and base fluid, and $\mu_r$ is the relative viscosity. Depending on the size and concentration of the solid particles, several models exist, that describe the relative viscosity as a function of volume fraction, $\phi$ of solid particles. Almost all the correlations have been derived from the pioneering work of Einstein [38], which is based on the assumption of a linearly viscous fluid containing dilute, suspended, spherical particles and low particle volume fraction ($\phi \leq 0.02$). Beyond this value, it underestimates the effective viscosity of the mixture. Considering non-interacting suspension, the suggested formula is as follows:

$$\mu_{nf} = \mu_{bf}(1 + 2.5\phi) \tag{6.17}$$

where, $\phi$ is the particle volume fraction in base fluid. This is a linear increase of the viscosity of the suspension with increasing solid volume fractions. Some factors have been identified as the limitations of this formula as the particle volume fractions may not be so small, the structure and the interactions of the particles within the continuous phase may also affect the viscosity of the mixture. Since the publication of Einstein's work, many researchers contributed to extend or correct this formula. In the case of higher concentration, a modified equation was proposed by Guth and Simha [39] which takes into account interaction between the solid particles and they defined $\mu_r = 1 + 2.5\phi + 14.1\phi^2$ so that the equation (6.16) becomes

$$\mu_{nf} = \mu_{bf}(1 + 2.5\phi + 14.1\phi^2) \tag{6.18}$$

Brinkman [16] proposed a more elaborated model that is currently used in the literature (Xuan and Roetzel [20], Khanafer et al. [40], Gosselin and Silva [41], Rahman et al. ([42], [44]), Rahman and Aziz [43], Rahman and Al-Hatmi [45]) for nanofluids. For particle concentrations less than 4%, Brinkman [16] formula is as follows:

$$\mu_{nf} = \mu_{bf}(1 - \phi)^{-2.5} \tag{6.19}$$

Krieger and Thomas [46] derived a semi empirical relation for the shear viscosity covering the full range of particle volume fraction:

$$\mu_{nf} = \mu_{bf}(1 - \phi/\phi_m)^{[\mu_i]\phi_m} \tag{6.20}$$

where, $\phi_m$ is the maximum particle packing fraction, which varies from 0.495 to 0.54 under quiescent conditions and, is approximately 0.605 at high shear rates and $[\mu_i]$ is the intrinsic viscosity, whose typical value for mono disperse suspensions of hard spheres is 2.5. Further modification of the equation of Guth and Simha [39] was proposed by Thomas [47] from the fitting of empirical data:

$$\mu_{nf} = \mu_{bf}(1 + 2.5\phi + 10.05\phi^2 + A \cdot e^{B \cdot \phi}) \tag{6.21}$$

where, $A = 0.00273$ and $B = 16.6$. Also, Frankel and Acrivos [48] presented a correlation of viscosity of suspensions using maximum particle volume concentration $\phi_m$ as determined by experiment which is as follows:

$$\mu_{nf} = \mu_{bf} \frac{9}{8} \left(\frac{\phi}{\phi_m}\right)^{\frac{1}{3}} / \left(1 - \left(\frac{\phi}{\phi_m}\right)^{\frac{1}{3}}\right) \tag{6.22}$$

Lundgren [49] proposed the following equation under the form of a Taylor series in $\phi$:

$$\mu_{nf} = \mu_{bf}(1 + 2.5\phi + 6.25\phi^2 + O(\phi^3)) \tag{6.23}$$

Considering the Brownian motion of particles on the bulk stress of an approximately isotropic suspension of rigid and spherical particles, Batchelor [17] proposed a following formula:

$$\mu_{nf} = \mu_{bf}(1 + 2.5\phi + 6.5\phi^2) \tag{6.24}$$

It is obvious from the above two relations that if the terms of the second or higher order of $\phi$ are neglected, the above relation reduces to that of Einstein's formula. Equation (6.24) has been validated for a particle volume fraction up to $\phi < 0.1$, where motion of single particle and pair particles interactions is dominant. Batchelor considered the interactions pair particles. In the case of high shear stress, another empirical equation was proposed by Kitano et al. [50] for polymer melts:

$$\mu_{nf} = \mu_{bf}(1 - \phi A^{-1})^{-2} \tag{6.25}$$

where, $A = 0.68$ for smooth spherical particles. Moreover, Graham [51] suggested a generalized form of the Frankel and Acrivos [48] formula including particle radius and inter particle spacing that is well in accordance with Einstein's formula for small $\phi$. His proposed formula is as follows:

$$\mu_{nf} = \mu_{bf}\left(1 + 2.5\phi + 4.5\left[1/(h(r_p)^{-1}(2 + h(r_p)^{-1})(1 + h(r_p)^{-1})^2)\right]\right) \tag{6.26}$$

Wang et al. [52] expressed a model to predict the viscosity of nanofluids as follows:

$$\mu_{nf} = \mu_{bf}(1 + 7.3\phi + 123\phi^2) \tag{6.27}$$

Chen et al. [53] showed a relation for the normalized shear viscosity with respect to the base fluid viscosity for up to 10% by nanoparticle volume fraction with an uncertainty less than 6%. They have also shown that relative viscosity increase is only a function of concentration and independent of temperature and their equation significantly deviates from the Einstein equation. They have given the following correlation by the regression of the experimental data as:

$$\mu_{nf} = \mu_{bf}(1 + 10.6\phi + 10.6\phi^2) \tag{6.28}$$

Nguyen et al. [54] stated that the formulas of both the Brinkman [16] and Batchelor [17] severely underestimate the nanofluid viscosities, except at very low particle volume fractions. Those linear fluid theories are very low comparing with their experimental results. They have proposed two correlations for nanofluids viscosities consisting 47 nm and 36 nm $Al_2O_3$ nanoparticles with water, considering the viscosities of base fluids and the particle volume fraction, respectively as follows:

$$\mu_{nf} = \mu_{bf} \times 0.904 e^{0.1482\phi} \tag{6.29}$$

$$\mu_{nf} = \mu_{bf}(1 + 0.025\phi + 0.015\phi^2) \tag{6.30}$$

They also have defined another correlation for computing the viscosity for $CuO$ - water nanofluids as

$$\mu_{nf} = \mu_{bf}(1.475 - 0.319\phi + 0.051\phi^2 + 0.009\phi^3) \tag{6.31}$$

Many scientist defined viscosity of nanofluids in terms of temperature. Kulkarni et al. [55] proposed a correlation of viscosity for $CuO$ - water nanofluids in the temperature range of $5°C - 50°C$ :

$$\ln\mu_{nf} = A(T^{-1}) - B \tag{6.32}$$

where, $A$ and $B$ are the function of volume fraction percentage $\phi$. It is clear from the equation that this equation is not applicable for subzero temperature. Abu-Nada [56] derived a correlation for nanofluid viscosity as a function of temperature and nanoparticle volume fraction. The experimental data reported by Nguyen et al. [54] have been used to derive this correlation. The expression is

$$\mu_{Al_2O_3} = -0.155 - \frac{19.582}{T} + 0.794\phi + \frac{2094.47}{T^2} - 0.192\phi^2 - 8.11\frac{\phi}{T} - \frac{27463.863}{T^3} + 0.127\phi^3 + 1.6044\frac{\phi^2}{T} + 2.1754\frac{\phi}{T^2} \tag{6.33}$$

This model has been compared with Brinkman model and found disagreement results. Furthermore, a modified mathematical model for calculating nanofluids viscosity with the help of exponential formula of Cheng and Law [57] and Einstein [38] has given by Avsec and Oblak [58] as

$$\mu_{nf} = \mu_{bf}(1 + 2.5\phi_e + (2.5\phi_e)^2) + (2.5\phi_e)^3 + \quad \cdots \quad \cdots \tag{6.34}$$

where, $\phi_e$ is the effective volume fraction.

## 6.3 Density and Specific Heat of Nanofluids

Calculation of the effective density and effective specific heat of nanofluids is straightforward. They can be estimated by the physical principle of the mixture rule and also, these results are in very good agreement with some experimental data. For these purpose, we define nanofluid as a mixture of consisting of a continuous base fluid and solid nanoparticles those are randomly moving in the fluid. That means $\rho_{nf} = (m/V)_{nf}$ implies the correlation of density as

$$\rho_{nf} = (1-\phi)\rho_{bf} + \phi\rho_p \tag{6.35}$$

The specific heat is the amount of heat per unit mass required to raise the temperature by one degree Celsius. The relationship between heat and temperature change is usually expressed in the form shown below where $C_p$ is the specific heat. The relationship does not apply if a phase change is encountered, because the heat added or removed during a phase change does not change the temperature. So, we define a mathematical equation as $Q = C_p m \Delta T$, where $Q$ is the amount of heat added, $C_p$ is specific heat, $m$ is mass and $\Delta T$ is the temperature difference. Thus, for nanofluid we can defined as $(\rho C_p)_{nf} = \rho_{nf}(Q/m\Delta T)_{nf}$ which turns to an equation $(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_{bf} + \phi(\rho C_p)_p$. Therefore, the specific heat of nanofluids can be defined as

$$(C_p)_{nf} = \frac{(1-\phi)(\rho C_p)_{bf} + \phi(\rho C_p)_p}{(1-\phi)\rho_{bf} + \phi\rho_p} \tag{6.36}$$

## 6.4 Electric Conductivity of Nanofluids

In metals, thermal conductivity approximately chases electrical conductivity according to the Wiedemann-Franz law, as freely moving valence electrons transfer not only electric current but also heat energy. However, the universal correlation between electrical and thermal conductance does not hold for other materials, because of the increased importance of phonon (vibrational energy) carriers for heat in non-metals. Jones and March [59] describes the Wiedemann-Franz law which is the ratio of the electronic contribution of the thermal conductivity ($\kappa$) to the electrical conductivity ($\sigma$) of a metal, and is proportional to the temperature $T$. That means,

$$\frac{\kappa}{\sigma} = LT \tag{6.37}$$

where, $L$ is a proportionality constant well-known as the Lorenz number, is equal to $2.44 \times 10^{-8} W\Omega K^{-2}$. The above relationship is based upon a statement that the heat and electrical transport both involve the free electrons in the metal. There are some strong limitations of Wiedemann-Franz law. Experiments reveal that the theoretical value of $L$ is roughly constant, is not exactly same for all materials. Kittel [60] presented some values of $L$ ranging from $2.23 \times 10^{-8}$ for copper at $0°C$ to $3.2 \times 10^{-8}$ for tungsten at $100°C$. Rosenberg [61] notes that the WiedemannFranz law is generally valid for high temperatures and for low temperatures, but may not hold at intermediate temperatures.

Since the electric conductivity and the thermal conductivity are correlated. It is expected that the electrical properties of nanofluids would satisfy the special requirements in practical applications. The study of the electrical properties of nanofluids would not only give a better understanding of transport mechanism for the system, but also widespread some potential applications for the nanofluids. It is true that the thermal conductivity and the electric conductivity are important properties of nanofluids. Although thermal conductivity of the nanofluids has been widely studied, electrical properties of nanofluid are studied insufficiently. Only few investigations have been carried out on the electric conductivity of carbon nanotube based nanofluids and very few data are available for nanofluids. In this respect, Lisunova et al. [62], Liu et al. [63] investigated the electric conductivity of aqueous suspensions of multi-walled carbon nanotubes and an exponential relation between the electric conductivity and the mass concentration of the carbon nanotube in aqueous suspension was noticed. Glover et al. [64] measured the electric conductivity of nanofluids based on functional sulfonated carbon nanotubes and found that the electric conductivity increases 13 times in nanofluid containing 0.5 wt % nanotubes. The electrical conductivity of the alumina nanoparticles in water was conducted by Ganguly et al. [65] and a linear dependence of electrical conductivity on volumetric concentration has been observed. The static electric conductivity are defined by Maxwell where, $\sigma$ depends on the phase volumetric fraction, $\phi$ as

$$\sigma_M = \sigma_{bf} \frac{2\sigma_{bf} + \sigma_p - 2\phi(\sigma_{bf} - \sigma_p)}{2\sigma_{bf} + \sigma_p + \phi(\sigma_{bf} - \sigma_p)} \tag{6.38}$$

where, $\sigma_{bf}$ and $\sigma_p$ are the electrical conductivity of pure fluid and the solid particle respectively. Shen et al. [66] shown that the measured critical conductivity of nanofluid is significantly larger than the expected value from the classical Maxwell model. That means the

Maxwell model underestimates the electrical conductivity enhancement in nanoparticle fluid mixtures. Therefore, it can be assumed that many dynamic factors are responsible for the enhancement of electrical conductivity in electrical transport process. The dynamic factor can be electrical double layer interactions, Brownian motion of the nanoparticles or even the agglomerations of nanoparticles. In many literatures, it can be suggested that electrical conductivity involves two separated transport process, one of which is static electrical conductivity while the other is dynamic electrical conductivity. The static conductivity is based on the Maxwell model which neglects the dynamic factors. For the dynamic conductivity, the electrophoresis and the Brownian motion of the particles are the key factors that play an important role in the electrical transport process.

The nanoparticle has a Zeta potential $U_0$ relative to the base liquid. When an electrical field is applied, the charged particles will move towards the electrode and thus form the electrophoretic conductivity, namely, the nanoparticles are one kind of the carriers in the fluid. For a spherical nanoparticle with diameter $d_p$, the electron attachment for the particle can be expressed as (see Dwyer [67])

$$q = 2\pi\varepsilon_r\varepsilon_0 d_p U_0 \tag{6.39}$$

According to the double layer theory, the total resulting force is zero, when the electric field is applied and the charged particle to be analyzed is at steady movement through the diffuse layer. Considering the drag on the moving particles due to the viscosity of the dispersant, in the case of low Reynolds number and moderate electric field strength $E$, the drift velocity of a dispersed particle, $V_d$ is simply proportional to the applied field, which leaves the electrophoretic mobility, $V_{em}$ defined as: $V_{em} = V_d/E$. Also, the most well-known and widely used theory of electrophoresis was developed by Smoluchowski [68] as

$$V_{em} = \frac{\varepsilon_r\varepsilon_0 U_0}{\mu_{bf}} \tag{6.40}$$

where, $\varepsilon_r$ is the dielectric constant of the dispersion medium, $\varepsilon_0$ is the permittivity of free space, $\mu_{bf}$ is dynamic viscosity of the dispersion medium, and $U_0$ is zeta potential (the electro kinetic potential of the slipping plane in the double layer). The Smoluchowski theory is very powerful because it works for dispersed particles of any shape at any concentration. Unfortunately, it has limitations on its validity. It follows, for instance, from the fact that it does not include Debye length $(\kappa^{-1})$. However, Debye length must be important for electrophoresis. Increasing thickness of the double layer (DL) leads to removing point of retardation force further from the particle surface. Detailed theoretical analysis proved that the Smoluchowski theory is valid only for sufficiently thin double layer, when particle radius, $r$ is much greater than the Debye length i.e. $r \gg \kappa^{-1}$. The Smoluchowski theory also neglects the contributions from surface conductivity. This is expressed in modern theory as condition of small Dukhin number (Dukhin number is a ratio of the surface conductivity to the fluid bulk electrical conductivity multiplied by particle size). In the effort of expanding the range of validity of electrophoretic theories, the opposite asymptotic case was considered, when Debye length is larger than particle radius: $r\kappa < 1$. Under this condition of a thick Double Layer, Huckel [69] predicted a relation for electrophoretic mobility as $V_{em} = \frac{2\varepsilon_r\varepsilon_0 U_0}{3\mu_{bf}}$. This electrophoretic mobility can be useful for some nanoparticles and non-polar fluids, where Debye length is much larger than in the usual cases. Hence, the electrophoretic conductivity due to electrophoretic mobility of nanofluids can be defined as $\sigma_E = V_{em}N_0 q = (4\pi d_p N_0 \varepsilon_r^2 \varepsilon_0^2 U_0^2)/(3\mu_{nf})$, where, $N_0$ is the number of particle in per unit volume of the nanofluids and can be define as $N_0 = (6\phi)/(\pi d_p^3)$. Hence, using this relation, we get

$$\sigma_E = \frac{8\phi\varepsilon_r^2\varepsilon_0^2 U_0^2}{d_p^2 \mu_{nf}} \tag{6.41}$$

In addition to the electrophoresis of the nanoparticle in base fluid, the Brownian motion of the nanoparticles in base fluid is also responsible to enhancing electrical conductivity of nanofluids. So, electric conductivity of nanofluids due to Brownian diffusion coefficient, $D_B$ (as calculated in section 10.4 equation (10.10) of the nanoparticles can obtain as

$$\sigma_B = D_B N_0 q = \frac{2\phi\varepsilon_r\varepsilon_0 U_0}{d_p^3} \frac{k_B T}{\pi\mu_{nf}} \tag{6.42}$$

Hence, we assume that the electric conductivity of nanofluids is stricken by the phase volume fraction, electrophoretic mobility and Brownian motion of nanoparticles. With regard to the above assumptions the new electrical conductivity equation for nanofluids can be proposed as

$$\sigma = \sigma_M + \sigma_E + \sigma_B = \sigma_{bf}\frac{2\sigma_{bf} + \sigma_p - 2\phi(\sigma_{bf} - \sigma_p)}{2\sigma_{bf} + \sigma_p + \phi(\sigma_{bf} - \sigma_p)} + \frac{8\phi\varepsilon_r^2\varepsilon_0^2 U_0^2}{d_p^2 \mu_{nf}} + \frac{2\phi\varepsilon_r\varepsilon_0 U_0}{d_p^3}\frac{k_B T}{\pi\mu_{nf}} \tag{6.43}$$

# 7  Applications of Nanofluids

Nanofluids are suspensions of nanoparticles in fluids that show significant enhancement of their properties at modest nanoparticle concentrations. Many of the publications on nanofluids about understanding their behavior so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. Notable publications on the applications of nanofluids are Wong and Leon [70], Choi [71], Saidur et al. [72], Yu and Xie [73], Taylor et al. [74], Mahian [75], etc. We will overview the applications of nanofluids as follows:

## 7.1  Biological and Biomedical Applications

Nanofluids have significant possibilities to contribute in biomedical applications such as drug delivery and nanofluids based control of biological functions because the size of the nanoparticles is similar to that of biomolecules. Gold nanoparticles are widely applied in the sensor field. Gold nanoparticles can serve as the surface modification material of DNA sensor to enhance their sensitivity. For example, gold nanoparticles are introduced into sensitive membrane preparation, the performance of chemical and biological sense will be greatly improved. Furthermore, nanotechnology has provided the possibility of delivering drugs to specific cells using nanoparticles. Most of the researches on bio- micro-electro-mechanical systems (MEMS) were done in the 1990s and recently commercialization of such strategies with technological equipment has started extensively. Various nanoforms of biological substances such as albumin, gelatin and phospholipids for vesicles and of chemical substances such as various polymers, nanoparticles in nanofluids have been tried to apply in drug delivery systems. The size of the nanoparticles are similar to biomolecules and can still be controlled within 100 nm, they can be very effectively used in these nanodrug delivery systems. These nanodelivery systems are designed to absorb or conjugate onto the particle surface. Hence, drugs can be protected from a critical environment or their unfavorable biopharmaceutical properties can be masked and replaced with the properties of nanomaterials. Furthermore, nanoparticles can be accrued preferentially at tumor, infectious sites and inflammatory places by the virtue of the enriched permeability and retention (EPR) effect. Based on those properties, nanodrug delivery systems offer many advantages. Shawgo et al. [76] included an example of an electronically activated drug delivery microchip. Immense literature and some strategies on nanodrug delivery system using nanofluids can be found in Moghimi et al. [77], Emerich and Thanos [78], Farokhzad and Langer [79].

## 7.2  Cancer Therapeutics

In cancer therapeutics, for the purpose of cancer imaging and drug delivery on the specific cells, the use of certain nanofluids is the new initiative due to its advantages of several properties. This initiative involves the use of iron-based nanoparticles as delivery vehicles for drugs or radiation in cancer patients. Magnetic nanofluids are used to guide the particles with bloodstream to a tumor with magnets. It will allow doctors to deliver high local doses of drugs or radiation without damaging nearby healthy tissue. Bica et al. [80] stated that magnetic nanoparticles are used because as compared to other metal type nanoparticles, these provide a characteristic for handling and manipulation of the nanofluid by magnetic force. This combination of targeted delivery and controlled release will also decrease the likelihood of systemic toxicity since the drug is encapsulated and biologically unavailable during transit in systemic circulation. Chiang et al. [81] described that nanofluid containing magnetic nanoparticles also acts as a super paramagnetic fluid which is an alternating energy producing a controllable hyperthermia. By enhancing the chemotherapeutic efficacy, the hyperthermia is able to produce a preferential radiation effect on malignant cells.

## 7.3  Antibacterial Agent

Nanofluids are using as an antibacterial agent. Antibacterial agents are very important in the textile industry, water disinfection, medicine, and food packaging. Organic compounds used for disinfection have some disadvantages, including toxicity to the human body. Therefore, the interest in inorganic disinfectants such as metal oxide nanoparticles (NPs) is increasing. Hajipour et al. [82] studied the properties and applications of inorganic nanostructure materials and their surface modifications, with good antimicrobial activity. Such improved antibacterial agents locally destroy bacteria, without being toxic to the surrounding tissue. They also provided an overview of opportunities and risks of using NPs as antibacterial agents.

## 7.4 Detergency

The possibilities of nanofluids are excellent candidates in the processes of soil remediation, lubrication, oil recovery and detergency because nanofluids have the same manner as simple liquids with classical concepts of spreading and adhesion on solid surfaces. Water is a fabulous cleaning agent, but it's even better when tiny nanoscale particles, such as detergent surfactants are dispersed in it. Much of this cleaning boost comes from the enhanced capacity of the detergent solution to spread out on surfaces. Wasan and Nikolov [83] speculated that the knowledge of the mechanism by which a particle-containing fluid spreads out and readily lifts oil droplets off a surface could improve the design of a variety of detergents.

## 7.5 Solar Industry

Solar energy is one of the best sources of renewable energy. Solar collectors are heat exchangers that convert solar radiation energy into internal radiation energy. These devices absorb the incoming solar radiation, convert into heat and transfer the heat to a fluid. The conventional solar collector is a well-established technology, and it has been proposed for a variety of applications such as water heating. However, the effectiveness of these collectors is limited by the absorption properties of the working fluid. Recently, this technology has been combined with the emerging technologies of nanofluids to create a new class of nanofluid based solar collectors. Tyagi et al. [84] investigated that by mixing nanoparticles to working fluids, the efficiency increases remarkably for the low values of volume fraction of nanoparticles. They investigated that nanoparticles have increased the collectors efficiency to the increase in reduction of sunlight passing through the collector. Otanicar et al. [85] investigated both experimental and numerical results on solar collectors based on nanofluids made from a variety of nanoparticles (carbon nanotubes, graphite, and silver). The efficiency improvement was up to 5% in solar thermal collectors by utilizing nanofluids as the absorption means. Under the similar operating conditions, the efficiency of an absorption solar collector using nanofluid as the working fluid was found to be up to 10% higher than that of a flat plate collector. Otanicar and Golden [86] showed that for the current cost of nanoparticles the nanofluid-based solar collector had a slightly longer profit period but at the end, it has the same economic saving as a conventional solar collector.

## 7.6 Industrial Cooling Applications

Nanofluids are using as a cooling objects in industry and have potential properties to be more useful in industrial cooling applications that could result in great energy savings and reductions of emissions. As for example, according to the Routbort et al. [87], it is possible to conserve one trillion Btu of energy using nanofluids in place of normal cooling and heating water. It is also predicted that for the U.S. electric power industry, using nanofluids in closed-loop cooling cycles could save about 10-30 trillion Btu per year. The associated emissions reductions would be approximately 5.6 million metric tons of carbon dioxide, 8,600 metric tons of nitrogen oxides and 21,000 metric tons of sulfur dioxide. In rubber and tire plants industry, nanofluids are a potential object to cooling the rubber efficiently during the processing time. At instant, the productivity of numerous industrial processes is constrained by the lack of facility to cool the rubber efficiently as it is being processed. In Michelin North America tire plants, there is a vision to obtain a 10% productivity increase in its rubber processing plants if suitable water-based nanofluids can be developed and commercially produced in a cost-effective manner.

## 7.7 Smart Fluid

A smart fluid is a fluid whose properties can be changed by applying an electric field or a magnetic field. Nanofluids have the potential ability to be used as smart fluids. In this new age of energy awareness, our lack of abundant sources of clean energy and the widespread dissemination of battery operated devices, such as cell-phones and laptops, cars, lights etc. have accented the necessity for a smart technological handling of energetic resources. Nanofluids have been demonstrated to be able to handle these roles in some instances as a smart fluid. Donzelli et al. [88] showed that a particular class of nanofluids can be readily constituted either in a low state, where it conducts heat poorly, or in a high state, where the dissipation is more efficient.

## 7.8 Nanofluids in Nuclear Reactors

In nuclear reactors, fossil fuel boilers, fusion reactors, nanofluids play an important role by increasing critical heat flux (CHF). Critical heat flux describes the thermal limit of a phenomenon where a phase change occurs during heating (such as bubbles forming on a

metal surface used to heat water), which suddenly decreases the efficiency of heat transfer, thus causing localized overheating of the heating surface. Kim et al. [89] and Buongiorno et al. [90] studied the feasibility of nanofluids in nuclear reactors and found possible applications. Buongiorno et al. [91] presented another potential use of nanofluids in nuclear systems. They postulated that nanofluids are the alleviation object for the postulated severe accidents during the core melts and relocates to the bottom of the reactor vessel. It is desirable to retain the molten fuel within the vessel by removing the decay heat through the vessel wall, if there are such accidents. This process is limited by the occurrence of CHF on the vessel outer surface, but investigation indicates that the use of nanofluid can increase the inner vessel retention capabilities of nuclear reactors by as much as 40%. Many water-cooled nuclear power systems are CHF-limited, but the application of nanofluid can greatly improve the CHF of the coolant so that there is a bottom-line economic benefit while also raising the safety standard of the power plant system.

## 7.9 Nanofluids as Superconductor

Nanofluids are superconductor. So, nanofluid can be used as a working fluid to extract energy from the earth core and processed in a pressurized water reactor (PWR) power plant system producing large amount of work energy. According to the report of MIT [92], over 2000 Zettajoule geothermal energy could be extracted using technological improvements. Currently only 200 Zettajoule geothermal energy would be extractable. For extracting energy from the outer layer of the earth which varies in length between 5 to 10 km and temperature between 5,00 to 1,000 degree Celsius, it is highly require to cooling down the pipes, machineries and equipment working in high friction and enormously high temperature environment. Nanofluids have the property to cooling down them.

In the sub-area of drilling technology, it is required to improve sensors and cooling electronics. Nanofluids are capable of improving sensors and operating electronics at higher temperature in down hole tools. There are revolutionary improvements of utilizing new methods of rock penetration cooled and lubricated by nanofluids will lower production cost. Such improvements will enable access to deeper, hotter regions in high grade formations or to economically acceptable temperatures in lower grade formations. Tran et al. [93] conducted research targeted at developing a new class of highly specialized drilling fluids that may have superior performances in high temperature drilling. They found that this research is applicable in high pressure and high temperature drilling, which may be crucial to functioning large quantities of previously unrecoverable domestic fuel resources.

## 7.10 Car Engine

Radiators are heat exchangers used for cooling internal combustion engines by circulating a liquid called engine coolant through the engine block, where it is heated, then through a radiator where it loses heat to the atmosphere, and then returned to the engine. The use of nanofluids as coolants would allow for smaller size and better positioning of the radiators. Owing to the fact that there would be less fluid due to the higher efficiency, coolant pumps could be shrunk and truck engines could be operated at higher temperatures allowing for more horsepower while still meeting stringent emission standards. In looking for ways to improve the aerodynamic designs of vehicles, and subsequently the fuel economy, manufacturers must reduce the amount of energy needed to overcome wind resistance on the road. At high speeds, approximately 65% of the total energy output from a truck is expended in overcoming the aerodynamic drag. This fact is partly due to the large radiator in front of the engine positioned to maximize the cooling effect of oncoming air. Argonne researchers, Singh et al. [94], have determined that the use of high-thermal conductive nanofluids in radiators can lead to a reduction in the frontal area of the radiator by up to 10%. This reduction in aerodynamic drag can lead to a fuel savings of up to 5%.

## 7.11 Nanofluid as Fuel

The use of nanofluid as fuel is a tremendous advancement in nanotechnology. It was shown that the combustion of diesel fuel mixed with aqueous aluminum nanofluid increased the total combustion heat while decreasing the concentration of smoke and nitrous oxide in the exhaust emission from the diesel engine. It is due to the high oxidation activity of pure Aluminum which allows for increased decomposition of hydrogen from water during the combustion process.

## 7.12 Nanofluid as Brake Fluid

Nanofluids are using as brake fluids. During the process of braking, the produced heat causes the brake fluid to reach its boiling point, a vapor lock is created that retards the hydraulic system from dispersing the heat caused from braking. It will create a brake malfunction

and poses a safety hazard in vehicles. Nanofluids with enhanced characteristics maximize performance in heat transfer as well as remove any safety concerns.

## 7.13 Microchips Electronic Industry

Nanofluid can be used for the cooling of microchips. In electronic industry, the quick heat dissipation is the main obstacle to developing smaller microchips. Ma et al. [95] presents the findings of a distinctive cooling device that makes use of an oscillating heat pipe (OHP) and nanofluids. It was also an innovation that has provided a way for cooling technology to keep pace with developments in electronic technology. It is expected that using this findings, the next generation of computer chips will produce localized heat flux more than 10 $MW/m^2$, with the total power exceeding 300 W. They also mentioned that there was no existing low cost cooling device that can effectively manage the heat production at this level. The study will accelerate the development of next generation of cooling devices that incorporate nanofluids for ultrahigh heat flux electronic systems. Even though nanofluids and OHPs are not new discoveries, combining their unique features allows for the nanoparticles to be completely suspended in the base liquid increasing their heat transport capability. Since nanofluids have a strong temperature dependent thermal conductivity and they show a nonlinear relationship between thermal conductivity and concentration, they are high performance conductors with an increased critical heat flux.

## 7.14 Nanofluid in Microscale Fluidic

In microscale fluidic applications, nanofluids are effective liquids for making wettability of the surface. In fluidic digital display devices, optical devices, and micro-electro-mechanical systems such as lab-on-chip analysis systems, the manipulation of small volumes of liquid is necessary. This can be done by electrowetting or reducing the contact angle by an applied voltage, the small volumes of liquid. Electrowetting on dielectric actuation is one very useful method of microscale liquid use. Vafaei et al. [96] investigated that nanofluids are very useful in controlling the wettability of the surface. It was observed from the above study that even the addition of a very low concentration of bismuth telluride nanofluid dramatically changed the wetting characteristics of the surface.

## 8  Heat Transfer Modeling on Nanofluids

Modeling of heat transfer in nanofluids is one of the key challenges to understand the behavior and applications of nanofluids. Currently, there are four varieties of heat transfer modeling in nanofluids.

## 8.1  Dispersion Model

This approach is based on the assumption that the convective heat transfer enhancement in nanofluids depend on two factors. One is higher thermal conductivity and another is stable dispersion of nanoparticles. Xuan and Roetzel [20] studied conceptions for heat transfer correlation of nanofluids. This was the first initiative in this approach. They discussed conventional approach for nanofluids modeling, and later proposed a dispersion model to handle the thermal dispersion resulting from irregular movement of the nanoparticles. They assumed that irregular movement of the ultrafine particles induces small perturbations of both the temperature and velocity of the nanofluids.

## 8.2  Particle Migration Modeling

Leighton and Acrivos ([97], [98]), Phillips et al. [99] and Liu [100] stated that three mechanisms are responsible for particle migration in non-uniform shear flows. These are (i) shear-induced migration where by particles move from regions of higher shear rate to regions of lower shear rate, (ii) viscosity gradient-induced migration where by particles move from regions of higher viscosity to those of lower viscosity and (iii) self-diffusion due to the Brownian motion through which particles move from regions of a higher concentration to those of a lower concentration. Wen and Ding [101] were the first to use these mechanisms of particle migration in nanofluids and developed a theoretical model to predict particle migration in pressure-driven laminar pipe flows of relatively dilute nanofluids.

## 8.3  Two Component Modeling

To investigate the heat transfer enhancement by nanoparticles suspension in a fluid, the popular two component approach takes into account the fluid phase and solid phase role in the heat transfer process and the absolute velocity of nanoparticles can be calculated as the sum of relative slip velocity and base fluid velocity. Buongiorno [23] first initiated this approach and developed an alternative two component model that has tried to eliminate the shortcomings of the homogeneous and dispersion models. He noted that heat transfer associated with nanoparticles dispersion is negligible compared with heat conduction and convection. In his new model, the effect of the nanoparticles and base-fluid comparative velocity is described more mechanistically than in the dispersion models. He has included nanoparticle continuity equation and emphasized that this equation must be coupled with the energy equation. Furthermore, he discussed seven slip mechanisms and finally showed that Brownian diffusion and thermophoresis may become important slip mechanisms among them. Many authors solved his mathematical model in several ways and implemented in different geometries as well as various flow conditions. Recently, Rahman and Eltayeb [102], Rahman et al. ([103] , [104], [105], [106]), Alam and Hossain [107] and Sheremet et al. [108] studied Buongiorno's two component model and found significant effects of Brownian motion and thermophoresis parameter.

## 8.4  One Component Modeling

In one component model, the traditional heat transfer equations for fluid dynamics are directly expanded to the nanofluids and the local velocity of nanofluids is same as the clear heat transfer fluids. This means that all the conventional heat transfer correlations could be used for nanofluids and also thermophysical properties of nanofluids are used in the calculations. Consequently, higher heat transfer is assumed to find out only from the enhanced thermal conductivity. This approach was initially conducted by Choi [1] and Maiga et al. [18]. Also, recently, Rahman and Aziz [43], Rahman et al. ([42] [44]), Rahman and Al-Hatmi [45] published results on one component model and found good agreements with the outcomes of Tiwari and Das [109].

Buongiorno [23] stated by the comparisons of analytical and experimental results that, to observe nanofluids heat transfer enhancement systematically, the homogeneous flow models are in conflict with the experimental investigations. However, Khanafer et al. [40] and Jou and Tzeng [110] emphasized on one component models and stated that in the absence of any suitable theoretical studies and experimental data in the literature to investigate these issues, the existing macroscopic two component model is not applicable for analyzing nanofluids. Furthermore, Tiwari and Das [109] presented that if the main interest is focused on the heat transfer process, the modified one component, accounting for some of the mechanisms, is more convenient than the two component model. They also noted that, the superior characteristics of the nanofluids allow it to behave more like a fluid rather than the conventional solid-fluid mixtures.

# 9  The Scales of Turbulent and Laminar Eddies

To justify the slip mechanisms effect on the nanofluids, it is important to know the time scale and length scale of laminar and turbulent eddies. From the Kolmogorovs first similarity hypothesis (Kolmogorov [111]), using a typical flow conditions i.e.$Re \sim 50,000$, $L \sim 1cm = 0.01m = 10^4 \mu m$, $U \sim 5m/s = 5 \times 10^6 \mu m$, we can calculate, the length and time scale of turbulent eddies as $L_{se} = LRe^{-3/4} = 2.99 \mu m$, $t_L = 2mis$ and $t_{L_{se}} = t_L Re^{-1/2} = 8.94 \mu s$ . Also, for laminar eddies i.e. considering $Re \sim 2300$, the length scale and time scale can be evaluated as, $L_{se} = LRe^{-3/4} = 30.11 \mu m$ and $t_{L_{se}} = t_L Re^{-1/2} = 41.70 \mu s$ .

# 10  Slip Mechanisms of Nanofluids

In order to predict nanofluids flow behavior more accurately, natural strikes such as rotation of nanoparticles, inertia, Brownian motion, thermophoresis, and gravity etc. can't be ignored. These natural strikes are known as slip mechanisms of nanofluids. Some of them are discussed below:

## 10.1 The Peclet Number in Nanofluids

Ahuja [112] experimented with micro-sized particles ($40 - 100 \ \mu m$) suspended in water and glycerin and demonstrated that particle rotation can create heat transfer enhancement. That is, under the effect of the shear stress, the suspended particles rotate about an axis perpendicular to the main flow direction, creating a three-dimensional hydrodynamic boundary layer which increases the fluid flow towards the wall. With respect to heat conduction, to evaluate the influence of this effect, Ahuja [112] proposed the use of a 'rotational' Peclet number (the ratio of the rate of advection of a physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient),

$$Pe_r \equiv \frac{V d_p}{\alpha} = \frac{V}{d_p} \frac{d_p^2}{\alpha} = \frac{\tau_w}{\mu} \frac{d_p^2}{\alpha} \tag{10.1}$$

where, $V$ is the velocity of the of the fluid, $\tau_w$ is the shear stress at the wall, $d_p$ is the particle diameter, $\mu$ is the fluid viscosity, and $\alpha = \kappa/(\rho c_p)$ is the fluid thermal diffusivity. In equation (10.1) the term $d_p^2/\alpha$ represents the time constant for conduction heat transfer, while the term $\tau_w/\mu$ represents the angular velocity of the particle, which Ahuja recommends assuming to be of the same order of magnitude of the shear rate at the wall. So, if $Pe_r$ is large, heat transfer enhancement by particle rotation is possible. The Darcy-Weisbach equations for channel flow are defined as (Hickin [113])

$$\tau_w \equiv \frac{f}{8} \rho \bar{V}^2 \tag{10.2}$$

where $\bar{V}$ is the mean axial velocity in the channel, $\rho$ is the fluid density, and $f$ is the Darcy-Weisbach friction factor. Using the definition of Reynolds number ($Re = \rho \bar{V} D / \mu$) and the friction factor correlation ($f = C/Re^n$) the equation (10.2) becomes

$$\tau_w = \frac{C}{8} \frac{\mu^2}{\rho D^2} Re^{2-n} \tag{10.3}$$

where, $D$ is the channel diameter. For room temperature the typical value of alumina/water nanofluids are $C = 0.184$, $\mu \sim 1 mPas$, $\rho = 1 g/cm^3$, $D = 1 cm$, $Re \sim 50,000$, $n = 0.2$, $\alpha = 2 \times 10^{-7} m^2/s$ and then from (10.3) we get $\tau_w = 6605015.54 mPa$ and for nanoparticles with $d_p < 100 nm = 1 \times 10^{-7}$, one gets $Pe_r < 0.0003$; therefore, we can decide that the rotation of nanoparticles can be chuck out as a heat transfer enhancement mechanism in nanofluids.

## 10.2 Calculation of Knudsen Number

For the purpose of these studies, it is assumed that the fluid around the nanoparticles can be regarded as a continuum. To assess the accuracy of the assumption, we need to calculate the Knudsen number ($Kn$), for kinetic theory, which is defined as the ratio of molecular mean free path length, $\lambda$ and representative physical length, $d_p$ . This length scale could be the radius of a body in a fluid. Water molecular mean free path can be calculated as: $\lambda = (n\sigma)^{-1} \approx 0.42 nm$, where the number density molecules of water, $n = 33.3679 \times 10^{21} cm^{-3}$, the collision cross section of water molecule $\sigma = \pi r^2 = 7.07 \times 10^{-16} cm^2$ with $r = 0.15 nm$. Hence, for nanofluids the Knudsen number ranges 0.42 to 0.0042 according to the nanoparticles diameter and the continuum assumption is reasonable.

## 10.3 Effect of Inertia

If any physical object does not change in its state of motion including changes to its speed and direction due to resistance and there is a tendency of objects to keep continuous moving in a straight line at constant velocity, this resistance is called inertia of object. As a result of inertia, when a particle suspended in a fluid could be developed a slip velocity in the presence of eddies (An eddy is the swirling of fluid and the reverse current created when the fluid flows past an obstacle).

The nanoparticles slip velocity can be obtained from the equation of motion $F = ma$ where the force of viscosity or the viscous resistance on a small sphere moving through a viscous fluid are calculated from the Stokes law and is given by $F_d = -6\pi\mu R V_e$, where $F_d$ is known as frictional force or Stokes drag acting on the interface between the fluid and the particles, the negative sign indicates that the drag force is acting in the direction opposing the particles motion, $\mu$ is the dynamic viscosity, $R = d_p/2$ is the radius of the spherical nanoparticle, $V_e$ is the nanoparticle slip velocity, $m = (4\pi R^3 \rho_p)/3$ is the mass of the nanoparticle with nanoparticle mass density ($\rho_p$) and $a = dV_e/dt$ is the acceleration of the nanoparticle. (Note that Stokes' law makes assumptions for the behavior of a

particle in a fluid that particles must be spherical, particles dispersion must be homogeneous with smooth surfaces, particles do not interpose each other and flow must be laminar ). So, from the equation of motion we get,

$$-6\pi\mu\left(\frac{d_p}{2}\right)V_e = \frac{4}{3}\pi\left(\frac{d_p}{2}\right)^3\rho_p\frac{dV_e}{dt} \Rightarrow \frac{dV_e}{V_e} = -\frac{18\mu}{d_p^2\rho_p}dt \tag{10.4}$$

From the above relation (10.4), we can evaluate the nanoparticle slip velocity as

$$V_e = V_{e0}e^{-(t/\tau_p)} \tag{10.5}$$

where, $\tau_p = (d_p^2\rho_p)/(18\mu)$ is the relaxation time (the return time of a perturbed particle to equilibrium) of the nanoparticles and $V_{e0}$ is the initial shear or friction velocity of the eddies. Considering typical values of alumina-water nanofluids where , $\mu \sim 1mPas$, $d_p < 100\ nm$ and $\rho_p \sim 4g/cm^3$, we get the relaxation time is $\tau_p < 2.2\ ns$ , which is negligible compared with the typical fluctuation time scale ($t_{L_{se}} = 8.94\ \mu s$) of the turbulent eddies. The nanoparticle stopping distance, $S_p$ can be obtained as

$$S_p = \frac{d_p^2\rho_p V_{e0}}{18\mu} \tag{10.6}$$

The shear velocity describes the diffusion and dispersion of the nanoparticles, tracers and contaminants in fluid flow and defined as

$$V_{e0} = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{f\rho\bar{V}^2}{8\rho}} = \bar{V}\sqrt{\frac{f}{8}} = \frac{\mu}{\rho D}\sqrt{\frac{C}{8}}Re^{1-\frac{n}{2}} \tag{10.7}$$

Now, again assuming values of $C = 0.184$, $\mu \sim 1mPas$, $\rho \sim 1g/cm^3$, $D \sim 1cm$, $n = 0.2$ and $Re \sim 50,000$ then from (10.7) and (10.6), we get $V_{e0} = 0.26\ mi/s$ and for nanoparticles with $d_p < 100\ nm = 1 \times 10^{-7}\ m$, one gets $S_p < 1\ nm$. Therefore, the stopping distance is much smaller than the length scale ($L_{se} = 2.99\mu m$) of the turbulent eddies. It made a clear idea that the movements of nanoparticles are homogeneous with the fluid in the presence of turbulent eddies. Also, the nanoparticles cannot enter into the laminar sub-layer near the wall by the virtue of their inertia. The thickness of the laminar sub-layer approximated as $\delta = 5(\frac{\mu}{\rho})(\frac{\tau_w}{\rho})^{-0.5}$. This is about $20\ \mu m$ for the typical value of the flow conditions. So, nanoparticle stopping distance much smaller than the laminar boundary layer thickness.

## 10.4 Brownian Diffusion in Nanofluids

In 1827, Robert Brown observed random motion of particles in pollen grains in water and noted that particles moved through the water but he was not able to determine the mechanisms that caused this motion. There were lots of theories about molecules and atoms which constituents of matter. Many decades later, Albert Einstein published a paper in 1905 that explained in precise detail how the motion that Brown had observed was a result of the pollen being moved by individual water molecules. Also, he described Brownian motion mathematically by Brownian diffusion coefficient using Stokes formula which is $D_B = k_B T/(6\pi\mu r)$ , where $k_B$ is the Boltzmanns constant, $T$ is the absolute temperature. So, the random motions of particles within the base fluid occur from the continuous collisions of particles and molecules of base fluids. After three years of Albert Einstein initiated modern study of random processes with his ground breaking paper on Brownian motion, in 1908, Paul Langevin, a French physicist and contemporary of Einstein, devised a very different but likewise successful description of Brownian motion. Both descriptions have since been generalized into mathematically distinct but physically equivalent tools for studying an important class of continuous random processes. Lemonsa and Gythiel [114] translated Langevin's paper and stated that Langevin's approach to Brownian motion is, in his own words, 'infinitely more simple' than Einstein's. Indeed, his paper is apparently simpler and for this reason is attractive as an introduction to the subject. While Einstein, starting from reasonable hypotheses and solved a partial differential equation governing the time evolution of the probability density of a Brownian particle, Langevin applied Newton's second law to a representative Brownian particle. In this way Langevin invented the $F = ma$ of stochastic physics called the Langevin equation. In short, Langevin gave a more direct treatment of Brownian motion. For understanding the Brownian diffusion coefficient, it is better to describe his process.

Let us assume that we are tracking a small spherical particle of mass $m$ and diameter $d_p$. It will experience a viscous drag force $F_d = -3\pi\mu d_p V$ (Stokes formula). We will denote $X$ is the random thermal collision force of particle which clearly averages to zero. The particles themselves can be viewed effectively as large molecules, with an average kinetic energy equal to that of the fluid molecules and thus with a considerable lower velocity, that is, $\frac{1}{2}m\bar{V}^2 = \frac{1}{2}k_B T$ , where average is over long time and hence $m\overline{\left(\frac{dx}{dt}\right)}$ . Now averaging

over a long time, the equation of motion $ma = F$ turns to $m \overline{\frac{d}{dt}\left(x\frac{dx}{dt}\right)} - m\overline{\left(\frac{dx}{dt}\right)^2} = -\frac{3}{2}\pi\mu d_p \overline{\frac{d}{dt}(x^2)} + \overline{Xx}$ . Since $X$ is random, $\overline{Xx} = 0$. The operation of averaging and taking the time derivative commute, we get, $\frac{m}{2}\frac{d^2}{dt^2}\overline{(x^2)} + \frac{3}{2}\pi\mu d_p \frac{d}{dt}\overline{(x^2)} = k_B T$. Let $\frac{d}{dt}\overline{(x^2)} = y$, the above differential equation then becomes $\frac{dy}{dt} + \frac{3\pi\mu d_p}{m}y = 2\frac{k_B T}{m}$. The solution of this differential equation is:

$$y(t) = \frac{2k_B T}{3\pi\mu d_p} + C\, e^{\left(-\frac{3\pi\mu d_p}{m}t\right)} \tag{10.8}$$

For the actual systems examined experimentally, the exponential term dies off in far less than a microsecond (approximately $10^{-8}$ seconds for the particles for which Brownian motion is observed, (see Lemons and Gythiel [114]). Hence,

$$\frac{d}{dt}\overline{(x^2)} = y(t) = \frac{2k_B T}{3\pi\mu d_p} \tag{10.9}$$

That means, $\overline{(x^2)} = \frac{2k_B T}{3\pi\mu d_p}t$. Using Einstein dynamical equilibrium formula for the Brownian diffusion coefficient for particles, we get,

$$D_B = \frac{\overline{(x^2)}}{2t} = \frac{k_B T}{3\pi\mu d_p} \tag{10.10}$$

For room temperature, $k_B = 1.38 \times 10^{-23}\, j/K$ , $\mu = 0.001\, pas$ , $T = 300K$ and with nanoparticles of $1 - 100\ nm$ diameter $(d_p)$, the Brownian diffusion coefficients ranges from $4 \times 10^{-10}$ to $4 \times 10^{-12}\ m^2/s$ . If $C$ is molar concentration then nanoparticles mass flux due to Brownian diffusion can be calculated as:

$$J_{p,B} = -D_B\nabla(\rho_p C) = -\rho_p D_B \nabla C \tag{10.11}$$

## 10.5 Thermophoresis Phenomena

The particle can diffuse from hotter region to colder region under the effect of temperature gradient. This phenomenon is called thermophoresis. It is not a result of thermal convection currents but is due entirely to the presence of a temperature gradient. This gradient causes a net imbalance in the average momentum transferred to a particle by the fluid molecules which surround it. This imbalance gives rise to a momentum flux, also known as the thermophoretic force, which moves the particle down the gradient with a terminal velocity called the thermophoretic velocity. Many scientists considered thermophoresis phenomena in liquids as well as gases because of the many similarities between two types of fluids. Thermophoresis in gas mixtures was first observed and reported by John Tyndall in 1870 when he observed that aerosol particles in a dust-filled room were driven away from heated surface and further understood by John Strutt (Baron Rayleigh) in 1882. For liquid mixtures, thermophoresis was first observed and reported by Carl Ludwig 1856 and further understood by Charles Soret in 1879.

Although our work is concerned with thermophoresis in nanofluids, a brief review of this phenomenon in gases and liquids is provided. This is organized for two reasons. First, the terminology and many important concepts, which have been well developed for gases, are also applicable to liquids. Seconds, the existing research on thermophoresis in nanofluids are not quite a significant level to provide an adequate understanding of this phenomenon.

### 10.5.1 Thermophoresis in Gases

Two regimes have been defined for the convenient analysis of thermophoresis. The definition is based on the limits of a single parameter, the Knudsen number $(Kn)$. The limits of $Kn$ corresponds to the small particle regime $(Kn \ll 1)$, sometimes called the free molecule region and the large particle region $(Kn \gg 1)$, also known as the continuum region. A zone exists between these two limits which ranges from approximately $0.1 < Kn < 10$ and is sometimes referred to as the transition region.

### 10.5.2 The small particle regime $(Kn \gg 1)$

In this region, the particles are considered to be so minute that they do not affect conditions in the gas. Molecule-particle interactions are infrequent and thus the number of reflected molecules in the vicinity of any one particle is very low. The equation predicting thermophoretic velocity was obtained from the kinetic theory. The net momentum transfer was computed by summing the momenta of the

incident molecules minus the momenta of the reflected molecules over the entire particle surface. Several models have been postulated for calculating the momenta of the reflected molecules but the principle ones are based on the elastic and diffusion collisions. Waldmann [115], Derjaguin and Bakanov [116], Mason and Chapman [117] derived independently the same equation for thermophoretic velocity in the small particle regime:

$$V_T = -\frac{6}{8+a\pi}\frac{\mu}{\rho}\frac{\nabla T}{T_k} \tag{10.12}$$

where $T_k$ and $\nabla T$ denote the absolute temperature and the fluid temperature gradient far away from the particle, respectively. By the definition, the coefficient $a$ is equal to zero for elastic (specular) molecule-particle collisions and when the directions of the reflected molecules are random.

### 10.5.3 The large particle regime $(Kn \ll 1)$

In this regime, the incident particles are influenced by the condition of the bulk of the fluid and by those near the particle surface. The kinetic theory approach employed for $Kn \gg 1$ cannot be used because of the simultaneous occurrences of the effect of the incident molecules. As a result a new model for the momentum transfer, partially based on continuum mechanics has been developed and is sometimes called the thermal creep (or slip) model. In this type of model, the mean momentum of the reflected molecules is a function of particle surface temperature and not the area of the fluid where they originated. Hence, upon collision, a molecule coming from the hot region gives momentum to the particle whereas a molecule from the cold region acquires momentum. Therefore, a net momentum flux exists and forces the particle down the temperature gradient. Maxwell [118] derived an equation for the thermal creep velocity as

$$V_s = C_s \frac{\mu}{\rho T_k}\frac{dT}{ds} \tag{10.13}$$

where, $C_s$ is a dimensionless parameter termed Maxwell's thermal creep coefficient whose value can also be established experimentally and $\frac{dT}{ds}$ is the tangential temperature gradient. The constitutive formula (10.13) holds independently of the particle size and shape, as well as of the particle's orientation relative to the temperature gradient, these being theoretical facts confirmed by experiment (Brenner [119]). Experiment further reveals that $C_s$ is unresponsive to the physiochemical constitution of either the gas or particle. Though Maxwell originally estimated his thermal creep coefficient to have the value, $C_s = 3/4$, gaseous thermophoretic experiments by Talbot et al. [120] cite the best fit value to be, approximately, $C_s \approx 1.17$. The thermophoretic velocity of Talbot et al. [120] has been used in many literatures such as Jayaraj et al. [121], Alam et al. ( [122], [123], [124]) and Alam [125]. Epstein [126] derived an expression for the thermophoretic velocity in the large particle regime. He solved the Fourier heat conduction equation to obtain the tangential temperature gradient along the surface of a (10.13) large particle and substitute this result into equation to obtain the thermal creep velocity. This velocity used as a boundary condition of the solution of the Navier-Stokes for creeping flow past a sphere. Epstein's final equation is

$$V_T = -\frac{3}{2}\left(\frac{\kappa_{bf}}{2\kappa_{bf}+\kappa_p}\right)\frac{\mu}{\rho}\frac{\nabla T}{T_k} \tag{10.14}$$

where $\kappa_{bf}$ and $\kappa_p$ are the fluid and particle thermal conductivities, respectively. Derjaguin et al. [127] and Keng and Orr [128] studied the Epstein's equation experimentally and established thermophoretic velocities within an order of magnitude, provided $Kn < 0.1$ and $\kappa_p/\kappa_{bf} \leqslant 10$. However, as $\kappa_p/\kappa_{bf} \to \infty$, this equation predicts $V_T \to 0$ whereas in reality, the velocity does not vanish. Brock [129] redefined Epstein's derivation by counting corrections for temperature jump (a function of the type of molecule particle collisions) and hydrodynamic slip as

$$V_T = -\frac{3}{2}\left(\frac{\kappa_{bf}+C_T\kappa_p Kn}{(2\kappa_{bf}+\kappa_p+2C_T\kappa_p Kn)(1+3C_M Kn)}\right)\frac{\mu}{\rho}\frac{\nabla T}{T_k} \tag{10.15}$$

where $C_T$ and $C_M$ are dimensionless constants accounting for the temperature jump and hydrodynamic slip, respectively. These constants are dependent on the chemical composition of the particle and fluid, and can only be determined from the experimental investigation. Brocks equation reduces to Epstein's equation when $Kn = 0$. Derjaguin and Bakanov [130] claimed that equation (10.13) is incorrect and added an additional heat flux term in the Fourier equation. Waldman and Schmidt [131] stated that this heat flux is poorly defined and neither their justification for this term nor their resulting derivation are well understood. Their final equation is

$$V_T = -\frac{1}{2}\left(\frac{8\kappa_{bf}+\kappa_p}{(2\kappa_{bf}+\kappa_p)}\right)\frac{\mu}{\rho}\frac{\nabla T}{T_k} \tag{10.16}$$

Derjaguin and Yalamov [132] introduced the temperature jump at the particle surface in the derivation of equation (10.16) and redefined as

$$V_T = -\frac{1}{2}\left(\frac{8\kappa_{bf}+\kappa_p+2C_T\kappa_p Kn}{(2\kappa_{bf}+\kappa_p+2C_T\kappa_p Kn)}\right)\frac{\mu}{\rho}\frac{\nabla T}{T_k} \qquad (10.17)$$

This equation has the same weakness regarding the experimental constant, $C_T$ as previously noted in the case of equation (10.15). When $Kn = 0$ , equation (10.17) reduces to equation (10.16). Derjaguin et al. [127] stated that both equation (10.16) and (10.17) predict velocities which are within an order of magnitude of the experimental values at $Kn < 0.1$. Neither equation predicts zero velocities at high $\kappa_p/\kappa_{bf}$. Fulford et al. [133] has been reported a preliminary study to verify the equations (10.14) to (10.17) for the use of temperature gradient to accelerate the deposition of small particles from laminar air streams. It is also shown that the acceptable and appreciable effects can be obtained even with small temperature gradients and particle diameter as large as 30 $\mu m$.

From the above observations, it can be noted that in all cases the thermophoretic velocity depend on the temperature gradient and if $Kn$ is very large or approximately zero, the thermophoretic velocity does not depend on particle diameter.

## 10.5.4 Thermophoresis in Liquids

Dwyer [134] attempted to establish a theoretical model for thermophoresis phenomena by modifying the derivation of isothermal equations for freely moving particles to include variable fluid temperature. He postulated from the Newton's second law of motion that the acceleration of one dimensional particle is equal to the frictional force term. This particular form of Newton's second law is sometimes called the Langevin equation.

Dwyer considered a small interval of time $\Delta\Theta$ , during which only the fluctuating term was allowed to vary. Dwyer did not include temperature gradient but defined temperature as a function of position and time because of the assumption of constant fluid temperature in each $\Delta\Theta$. He used this distribution to obtain an equation for the mean square displacement, $\langle \Lambda^2 \rangle$, in one dimension after time $\Theta$. Dwyer's equation is:

$$\langle \Lambda^2 \rangle = \langle \frac{2k_B}{\beta m_p}\int_0^\Theta T_k\Lambda(\xi)(1-exp(\beta(\xi-\Theta)))^2 d\xi \rangle \qquad (10.18)$$

where, $\beta = (3\pi d_p\mu)/m_p$ and $k_B$ is the Boltzmann's constant and $m_p$ denotes the particle mass. The probability averaging operator is symbolized by $\langle\ \rangle$ and all identical particles starting at the same initial temperature. He also mentioned two particular cases of varying temperature. In one case, he defined temperature is a linear function of displacement, i.e. $T_k = T_{k0} + N\Lambda$, where $\Lambda = x' - x_0'$, $T_{k0}$ is the temperature at the initial point, $x_0'$ and $N$ is a constant temperature gradient. Dwyer replaced equation $T_k = T_{k0} + N\Lambda$ into equation (10.18) and observed that a constant temperature gradient does not change the mean square displacement from that of the isothermal $(T_{k0})$ value, i.e., there is no thermophoresis for a constant temperature. He claimed that the square of the particle displacement is increased by the same amount when the particle goes up the gradient as it is decreased when the particle goes down the gradient. In the averaging procedure, this difference vanishes. In another case, he defined the temperature is a linear function of the square of the displacement i.e. $T_k = T_{k0} + M\Lambda^2$. If this equation use in equation (10.18), the mean square displacement will increase when $M$ is positive and decrease if $M$ is negative. Dwyer explained this by saying that a positive $M$ indicates that particles will always move to the region of higher temperature from its initial point no matter what direction it takes, while on the contrary, a negative $M$ means that particle will always travel into colder regions. With $T_k = T_{k0}$ , the equation (10.18) becomes upon integration:

$$\langle \Lambda^2 \rangle = \langle \frac{k_B T_{k0}}{\beta^2 m_p}\Theta(2\beta - \frac{3}{\Theta} + \frac{exp(-\beta\Theta)}{\Theta}(4-exp(-\beta\Theta)))\rangle \qquad (10.19)$$

As $\Theta$ approaches infinity, equation (10.19) tends to

$$\langle \Lambda^2 \rangle = 2\frac{k_B T_{k0}}{\beta m_p}\Theta = \frac{2k_B T_{k0}}{3\pi\mu d_p}\Theta \qquad (10.20)$$

Equation (10.20) is identical to Einstein equation for one dimensional mean square displacement in a uniform fluid. It indicates that the steady state part of equation (10.19) is correct. McNab and Meisen [135] speculated that the validity of transients in Dwyer's equation has not been established. They also stated that although Dwyer uses equation (10.18) to supports thermophoresis in liquids, each of his derivational steps is equally true for gases and Dwyer's equation is incompatible with the experimental investigations. McNab and Meisen [135] had established thermophoretic velocity and thermophoretic force experimentally using spherical polystyrene latex particles in water and n-hexane $C_6H_{14}$. The size of the particles were extremely uniform and two diameters were selected for their work, $d_p = 1011\ nm$ and $d_p = 790\ nm$ . They adopted 1.05 as the specific gravity and $\kappa_p = 0.128$, the thermal conductivity of the particle. Furthermore, they considered viscosity, density and thermal conductivity for water and hexane accordingly.

For water:

viscosity is defined as, $\frac{1}{\mu} = 21.482((T_c - 8.435) + (8078.4 + (T_c - 8.435)^2)^{1/2}) - 1200$ in the temperature range of interest $5°C \leq T_c \leq 90°C$, density is defined as, $\rho = 999.168 - 0.00426T_c^2$ with $15°C \leq T_c \leq 95°C$ and the thermal conductivity correlation is considered as, $\kappa_{bf} = 0.567558 + 0.001862289T_c - (7.15006 \times 10^{-6})T_c^2$

For hexane:

viscosity, $\mu = 0.00117 - 0.0000029T_k$ , where $T_k$ is the temperature expressed in Kelvins, density, $\rho = 931.5 - 0.928T_k$ and thermal conductivity, $\kappa_{bf} = 0.140 - 0.00044T_c$ . According to this equation, hexane has a thermal conductivity that is approximately $1/5$ to $1/4$ that of water. They defined thermophoretic velocity as

$$V_{th} = -\alpha \frac{\mu}{\rho} \frac{\Delta T}{T_k} \tag{10.21}$$

where the proportionality factor (dimensionless coefficient), $\alpha$ equals 0.1148 for water and 0.0874 for hexane. Previously, Epstein (equation (10.14)) and Derjaguin (equation (10.16)) defined $\alpha$ for gases where Knudsen number is equal to zero as follows:

$$\textbf{Epstein}[126] \qquad : \qquad \alpha = \frac{3}{2} \frac{\kappa_{bf}}{2\kappa_{bf} + \kappa_p} \tag{10.22}$$

$$\textbf{Derjaguin and Bakanov}[130] \qquad : \qquad \alpha = \frac{1}{2}\left(\frac{8\kappa_{bf} + \kappa_p}{2\kappa_{bf} + \kappa_p}\right) \tag{10.23}$$

Using the data of $\kappa_{bf} = 0.6000$ for water, $\kappa_{bf} = 0.130$ for hexane, $\kappa_p = 0.128$ , the values of $\alpha$ for hexane and water by the equations of (10.22) and (10.23) are displayed in the table.

| Author | $\alpha$ (water) | $\alpha$ (hexane) | $\alpha$ (hexane)/$\alpha$ (water)% |
|---|---|---|---|
| Epstein [126] | 0.6777 | 0.5026 | 74.2 |
| Derjaguin and Bakanov [130] | 1.855 | 1.505 | 81.2 |
| McNab and Meisen [135] | 0.1148 | 0.0874 | 76.1 |

As can be seen, the ratio for liquids is very close to that predicted for gases by the Epstein's relationship. McNab and Meisen stated that although essentially only two values of liquid thermal conductivity have been studied, it appears that an expression similar to Epstein's relation holds for both types of fluid. Hence, they defined a relationship among $\alpha$, $\kappa_{bf}$ and $\kappa_p$ as $\alpha = \gamma \frac{\kappa_{bf}}{2\kappa_{bf} + \kappa_p}$, where $\gamma$ can be calculated from the above data as $\gamma$(water) = 0.254 and $\gamma$(hexane) = 0.261. Since only two figures can be regarded as significant, $\gamma$ becomes 0.26 for both fluids. Therefore, by McNab and Meisen [135], an empirical thermophoretic velocity equation for liquid which is independent of particles diameters and is a function of liquid thermal conductivity is as follows:

$$V_{th} = -0.26\left(\frac{\kappa_{bf}}{2\kappa_{bf} + \kappa_p}\right)\frac{\mu}{\rho}\frac{\nabla T}{T_k} \tag{10.24}$$

The thermophoretic force, $F_{th}$ can be obtained from the thermophoretic velocity by means of Strokes law. The Strokes drag force, $F_D = 3\pi\mu d_p v_p$, where $v_p$ is the speed of the particle relative to the stagnant fluid. At steady state, this particle velocity is equal to $V_{th}$ and $F_{th} = -F_D$. Therefore using thermophoretic velocity equation, the thermophoretic force is

$$F_{th} = -0.78\pi\left(\frac{\kappa_{bf}}{2\kappa_{bf} + \kappa_p}\right)\frac{d_p\mu^2}{\rho}\frac{\nabla T}{T_k} \tag{10.25}$$

Brenner [136] stated that equation (10.24) is identical to the Epstein theoretical formula (10.14) for the thermophoretic velocity of a heat conducting spherical particle in a gas, where Maxwell's slip coefficient $C_s$ (multiplied by a factor of 2) appears in place of $\gamma$ (see equation (10.14)). With regard to its application to liquids equation (10.24) is completely empirical despite its theoretical basis for the case of gases. He questioned on these equations and analyzed them with the dilute mixtures of polystyrene latex particle in water at $25°C$ via the relation, $V_{th} = -D_T(0)\nabla T$, where $D_T(0) = (\alpha\mu)/(\rho T_k)$. Using the values of $\alpha = 0.1148$, kinematic viscosity $\nu = \mu/\rho = 8.95 \times 10^{-7}m^2/s$ and $T_k = 298°K$, thermal diffusivity coefficient can be calculated as

$$D_T(0) = 345 \times 10^{-12}m^2 s^{-1} K^{-1} \tag{10.26}$$

Experimental thermal diffusion data from several independent sources are available for dilute water-polystyrene latex particle mixtures at room temperature, and for several particle sizes. Thus for 60 nm diameter polystyrene latex particles and a temperature of $25°C$, Iacopini et al. [137] found that

$$D_T(0) = 1.5 \times 10^{-12} m^2 s^{-1} K^{-1} \tag{10.27}$$

On the other hand for the same mixture, Shiundu et al. [138] using thermal field flow fluctuation techniques observed the following values

| Particle diameters (nm) | Thermal diffusivity $D_T(0) \times 10^{12} (m^2 s^{-1} K^{-1})$ |
|---|---|
| 105 | 3.63 |
| 220 | 3.05 |
| 300 | 2.55 |
| 398 | 2.01 |

Brenner [136] manifested the existence of self-thermophoresis, a phenomenon whereby an effective thermophoretic force developing by a temperature gradient in a dormant single-component liquid or gas acts upon an individual molecule of that fluid in much the same manner as a "real" thermophoretic force acts upon a macroscopic, non-Brownian body immersed in that same fluid. Successively, self-thermophoresis acting together with Brownian self-diffusion gives rise to the phenomenon of thermal self-diffusion in single-component fluids. Explicitly, the self-thermophoretic theory furnishes a simple expression for both the thermophoretic velocity, $V_{th}$ of a macroscopic body in a single component fluid subjected to a temperature gradient, $\nabla T$ and the intimately related constituent thermal diffusion coefficient, $D_T$ for a two-component colloidal or macromolecular mixture. The predicted expression is

$$V_{th} = -D_T \nabla T = -\beta D_s \nabla T \tag{10.28}$$

where, $D_T = \beta D_s$, $\beta$ is the pure solvent's respective thermal expansion and $D_s$ is the isothermal self-diffusion coefficients. They are each noted to accord reasonably well with experimental data for both liquids and gases. Also, Brenner [119] presented a table, with the properties of liquid water at 1 atm and various temperatures used to calculates the hypothetical thermal diffusivity $(D_T^*)$ of solutes dissolved or dispersed in water. By the way of comparison, Brenner [136] calculated

$$D_T(0) = 0.6 \times 10^{-12} m^2 s^{-1} K^{-1} \tag{10.29}$$

where $D_s = 2.30 \times 10^{-9} m^2 s^{-1}$ and $\beta = 0.257 \times 10^{-3} K^{-1}$ at $25°C$. He concluded that equation (10.27) and (10.29) are consistent with one another. Unfortunately, these relations are in major disagreement with the thermal diffusion coefficient (equation (10.26)) calculated by McNab and Meisen [135]. Thus, because of the various approximations entered into the equation, $D_T(0) = \beta D_s$, he compensated for possible inaccuracies by inserting an empirical correction factor into the relation and proposed thermal diffusion coefficient as

$$D_T = -\lambda \beta D_s \tag{10.30}$$

where, the dimensionless parameter $\lambda$ is expected to be of $O(1)$ for both liquid and gases.

## 10.5.5 Thermophoresis in Nanofluids

We have seen that the theory and the experiment of thermophoresis in liquids are far from well-established even in the simplest case of a neutral particle in a non-conducting liquid. First of all, there are far fewer studies on the subject matter. The major difficulty lies between the facts that there may some mechanism responsible for the thermophoretic flow other than the kinetic interaction. The mechanism may be closely related to the liquid state in the interfacial region whose characteristics resist the usual theoretical or experimental approaches. May be the thermal volumetric fraction of the particles and the thermal expansion of fluids are responsible for discrepancy. The coefficient of thermal expansion describes how the size of an object changes with a change in temperature. Specifically, it measures the fractional change in size per degree change in temperature at a constant pressure. The volumetric thermal expansion coefficient is the most basic thermal expansion coefficient, and the most relevant for fluids. In general, substances expand or contract when their temperature changes, with expansion or contraction occurring in all directions. Substances that expand at the same rate in every direction are called isotropic. For isotropic materials, the area and volumetric thermal expansion coefficient are, respectively, approximately twice and three times larger than the linear thermal expansion coefficient.

Thermophoretic behaviors in liquids and gases are most important study. As particles are main concern for nanofluids and particles are affected by the temperature gradient, it is noticeable to consider thermophoretic velocity for describing the performances of nanofluids. It is a matter of great concern for the author's best knowledge that there is no thermophoretic velocity equation for nanofluids in literature. However, Buongiorno [23] explained nanofluids using the thermophoretic velocity equation of McNab and Meisen [135], where he experimented with the polystyrene latex particles of size 1011 nm and n-hexane particles of size 790 nm. Since, nanofluids are defined by the particle sizes of 1-100 nm; our aim is to establish a new correlation of thermophoretic velocity equation within the range of particle size of nanofluids with the close agreement of experimental results. We assume that thermophoretic velocity of nanofluids depend on the thermal expansion of bulk fluid, viscosity, density, and thermal conductivity of nanofluids as well as size of the particles. Therefore, considering the above factors, *the new thermophoretic velocity equation* for nanofluids can be proposed as

$$V_T = -0.21\kappa_{nf}\frac{\hbar\beta\mu_{nf}}{\rho_{nf}}\nabla(\ln T) \tag{10.31}$$

In equation (10.31), $\kappa_{nf} = \frac{2\kappa_{bf}+\kappa_p-2\phi(\kappa_{bf}-\kappa_p)}{2\kappa_{bf}+\kappa_p+\phi(\kappa_{bf}-\kappa_p)}\kappa_{bf}$, $\mu_{nf} = \mu_{bf}(1-\phi)^{-2.5}$, $\kappa_{bf}$ is thermal conductivity of base fluid, $\kappa_p$ is thermal conductivity of nanoparticle, $\phi$ is the volumetric fraction of the nanofluids, $\beta$ is the thermal expansion of the bulk fluid, $\mu_{bf}$ represents the dynamic viscosity of base fluid, $\rho_{bf}$ is the density of base fluid, $\rho_p$ is the density of the nanoparticle and the symbol $\hbar$ is the correction factor which depends on the size of the particles and followed a correlation $\hbar = -0.0002d_p + 0.1537$ . For particle size of 100 nm, the values of $\hbar$ has been found 0.1337 and for the particle size of 1 nm the value of $\hbar$ is 0.1535. The coefficient 0.21 has been found using McNab and Meisen [135] calculation process. In a similar manner, the proportionality factor, $\alpha$ can be redefined with new thermal conductivity correlation as

$$\alpha = \omega\kappa_{bf}\frac{2\kappa_{bf}+\kappa_p-2\phi(\kappa_{bf}-\kappa_p)}{2\kappa_{bf}+\kappa_p+\phi(\kappa_{bf}-\kappa_p)} \tag{10.32}$$

where, $\omega$ can be obtained as $\omega$ (water) = 0.1148/0.4805 = 0.24 and $\omega$ (hexane) = 0.0874/0.4805 = 0.182 So average value 0.21 may be regarded as the value of $\omega$ . The correlations of thermal conductivity, viscosity and density can be found in different studies such as Maxwell-Garnett's (MG model), Brinkman [16] and Xuan and Li [21]. Hence, the thermal diffusion coefficient can be incurred from the equation (10.31) as

$$D_T = 0.21\kappa_{nf}\frac{\hbar\beta\mu_{nf}}{\rho_{nf}} \tag{10.33}$$

At $298°K$ using the values of $\kappa_{bf} = 0.6000\ Jm^{-1}s^{-1}K^{-1}$, $\kappa_p = 0.128\ Jm^{-1}s^{-1}K^{-1}$, $\phi = 0.05$, $\rho_{bf} = 1000\ kgm^{-3}$, $\rho_p = 4000\ kgm^{-3}$, $\mu_{bf} = 8.973 \times 10^{-4} pa.s$, $\beta = 0.257 \times 10^{-3}K^{-1}$ and $\hbar = 0.1337$, thermal diffusion coefficient turns

$$D_T = 3.639 \times 10^{-12}\ m^2s^{-1}K^{-1} \tag{10.34}$$

Hence, we can conclude that our derived equation (10.34) is in excellent agreement with the experimental work of Shiundu et al. [138]. The nanoparticles mass flux due to thermal diffusion can be calculated as

$$J_{p,T} = -D_T(\rho_p C)\nabla(\ln T) \tag{10.35}$$

where, $C$ is the molar concentration of nanofluids.

Therefore, using the equation (10.11) and the equation (10.35), the total mass flux due to Brownian diffusion and thermal diffusion can be calculated as

$$J = J_{p,B} + J_{p,T} = -D_B\nabla(\rho_p C) = -\rho_p D_B\nabla C - D_T(\rho_p C)\nabla(\ln T) \tag{10.36}$$

So, our derived thermophoretic velocity equation (10.31), thermal diffusion coefficient equation (10.33) and total mass flux equation (10.36) can be used for formulating the general transport model in nanofluids. These equations are new and cannot be found in any open literature.

## 10.6 Terminal Velocity of Nanoparticles Due to Gravity

Sedimentation or settling is the process by which particulates fall slowly to a stable position, usually on a surface or to the bottom of a liquid and then form sediment. Particles are considered individually for settling. There are two main forces enacting upon any particle. The primary force is an applied force such as gravity and another one is drag force due to the motion of the particles through the fluid. Nanoparticle settling velocity due to gravity, $V_g$ can be calculated from a balance of the gravitational buoyancy and drag forces on the

submerged spherical solid body, where again Stokes law was used for the viscous resistance. From the universal law of drag force, we can write, $m_p g - m_{bf} g = F_d \Rightarrow (\pi d_p^3 (\rho_p - \rho_{bf}) g)/6 = 3\pi \mu_{bf} d_p V_g \Rightarrow V_g = d_p^2 (\rho_p - \rho_{bf}) g/(18\mu_{bf})$ i.e. for $d_p \leq 100\,nm$, $g = 9.8 ms^{-2}$, $\rho_p = 4 g/cm^3$, $\rho_{bf} = 1 g/cm^3$ and, $\mu_{bf} = 1 g/ms$, the nanoparticle settling velocity, $V_g < 1.63 \times 10^{-8} m/s$ . Thus, with respect to the reference length of the nanoparticle and the medium of the investigation, the gravitational effect can't be overlooked and it must be taken into account along with thermophoresis and Brownian diffusion.

## 11 Conclusions

Many interesting properties of nanofluids have been reported in the past decades. This study presents an overview of the recent works and basic understanding of nanofluids, including a brief historical outline of nanofluid research, the clear idea about the nanoparticles and the preparation methods of nanofluids. The thermal conductivity, viscosity, specific heat and electric conductivity of nanofluids have been analyzed and a comprehensive mathematical equation of electrical conductivity for nanofluids has been proposed. The broad applications of nanofluids have been included in the present literature. Brief descriptions on nanofluid modeling such as dispersion modeling, particle migration modeling, two component modeling and one component modeling have also enclosed. Some significant nanofluids slip mechanism with scales of turbulent and laminar eddies have reported in the present study. The equations of thermophoresis for gases and liquids have overviewed and with the justification of experimental data, the mathematical equations of thermophoretic velocity and the thermal diffusion coefficient for nanofluids as well as nanoparticles mass flux equation due to Brownian diffusion and thermal diffusion have been proposed newly for developing general transport nanofluid modeling.

It is important to note that the nanofluid research is getting more and more active and important. Applications of nanofluids are promising and they can be used in numerous sectors involving heat transfer and other cooling applications. In biological and biomedical sectors, nanofluids are playing important roles for a long time and their use will extend to growth. Nanofluids have also been manifested for use as smart fluids and detergents. In fossil fuel boilers, cancer therapeutics, electronic industries, mechanical industries, solar sectors, nuclear reactors and fusion reactors nanofluids are an important tool of getting expected results. In experimental research, nanofluids have to be well characterized with respect to particle size, particle shape and clustering so as to provide the results most extensively applicable. Once the experimental and theoretical investigations of nanofluids are fully understood with potential agreements, they can be reproduced on a huge scale and used in many applications.

The preparation of nanofluids is an important step for investigating the properties of nanofluids. The stability of the suspensions is a crucial issue for both scientific research and practical applications. The long term stability of nanofluids in the practical conditions should be paid more awareness. Having successfully engineering the nanofluids, the estimation of thermophysical properties of nanofluids captures the attention. Most of the thermal conductivity correlations of nanofluids have been postured with the classical diffusion approach with alteration for liquid layering, particle aggregation, or interfacial thermal resistance and the hydrodynamic approach to integrate new nanoscale convection. Although most of the models declared reasonable agreement with the experimental data, one or two empirical constants are used to reproduce the data. In the proposed thermal transport modeling, numerical simulations such as molecular dynamics are essential to realize the origin and the nature of the adjusting parameters. Also, each of the many assumptions and concepts used in new models need a fundamental proof. Therefore, controlled experiments are required to construct the validity of the thermal transport models at small length and time scales. In the heat and mass transfer nanofluids modeling, specific and precise slip mechanisms are vital to describe the flow patterns. Otherwise, the results sometimes violate the agreement with the experimental outcomes. Brownian motion and thermophoresis are two important slip mechanisms for nanofluid research and there should be careful investigation.

As synthesis, stability, modeling, and application are the essential parts of nanofluids research, further investigations still have to be done on them so that they may be applied as predicted. Nevertheless, in past decades, there have been many breakthroughs and improvements distinguished about the characteristics of nanofluids and we are one step closer to reach our goal that are more efficient for our practical applications.

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