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Pseudo almost periodic solutions for a Nicholson's blowflies model with mortality term*

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Abstract. This article is concerned with a discrete Nicholson's blowflies model, which involves a nonlinear density-dependent mortality term. By using fixed point theorem and Lyapunov functional method, we obtain the existence and locally exponential stability of pseudo almost periodic solutions for the addressed Nicholson's blowflies model. In addition, an example is given to illustrate our results.

Key words: pseudo almost periodic, Nicholson's blowflies model, exponential stability.

1 Introduction and preliminaries

In population dynamics, the classical Nicholson's blowflies equation developed by Gurney et al. [12] takes the following form:

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$$x'(t) = -\alpha x(t) + \beta x(t-\tau)e^{-\gamma x(t-\tau)},$$

where $x(t)$ denotes the population of sexually mature adults at time t, β is the maximum possible per capita daily egg production rate, $\frac{1}{\gamma}$ is the size at which the population reproduces at its maximum rate, α is the per capita daily adult death rate, and τ is the generation time.

Recently, the study on Nicholson's blowflies model has attracted much attention. Especially, several authors have made contribution on the existence of periodic solutions and almost periodic type solutions(see, e.g., [14, 15, 18, 9, 2, 3, 1, 21, 17, 5, 6, 11, 16, 19, 10] and references therein). Especially, in [16, 19, 10], the authors investigated the existence and stability of almost periodic solutions for Nicholson's blowflies models with a nonlinear density-dependent mortality term:

$$x'(t) = -\alpha(t) + b(t)e^{-x(t)} + \beta(t)x(t-\tau(t))e^{-\gamma(t)x(t-\tau(t))}. \tag{1.1}$$

In fact, as pointed out by Berezansky et al. in [4], a new study indicates that a linear model of density-dependent mortality will be most accurate for populations at low densities, and marine ecologists are currently in the process of constructing new fishery models with nonlinear density-dependent mortality rates. Therefore, studying the dynamical behavior for Nicholson's blowflies models with a nonlinear density-dependent mortality term is an interesting and important topic.

In this paper, we aim to study the existence and stability of pseudo almost periodic solutions for the following discrete Nicholson's blowflies model given by

$$\Delta x(n) = -\alpha(n)x(n) + b(n) + \beta(n)\ln x(n-\tau(n)) \cdot \frac{x(n)}{x^{\gamma(n)}(n-\tau(n))}, \tag{1.2}$$

where $n \in \mathbb{Z}$ and α , b , β , γ , $\tau : \mathbb{Z} \to [0,+\infty)$ are pseudo almost periodic sequences.

Eq. (1.2) can be seen as a discrete analogue of Eq. (1.1). In fact, letting $x(t) = \ln y(t)$ in Eq. (1.1), one can obtain

$$y'(t) = -\alpha(t)y(t) + b(t) + \beta(t)\ln y(t-\tau(t)) \cdot \frac{y(t)}{y^{\gamma(t)}(t-\tau(t))}.$$

Throughout this paper, we denote by $\mathbb{R}$ the set of real numbers, by $\mathbb{R}^+$ the set of nonnegative real numbers, by $\mathbb{N}$ the set of positive integers, by $\mathbb{Z}$ the set of integers, and by $\mathbb{Z}^+$ the set of nonnegative integers. Moreover, for every bounded sequence $f : \mathbb{Z} \to \mathbb{R}$, we denote

$$f^+ = \sup_{n\in\mathbb{Z}} f(n), \quad f^- = \inf_{n\in\mathbb{Z}} f(n).$$

Next, let us recall some basic definitions and results about almost periodic type sequences. For more details, we refer the reader to [7, 20, 8].

Definition 1. A set $E \subset \mathbb{Z}$ is called relatively dense if there exists $l \in \mathbb{N}$ such that

$$[n, n+l] \cap \mathbb{Z} \cap E \neq \emptyset$$

for every $n \in \mathbb{Z}$.

Definition 2. A function $f : \mathbb{Z} \to \mathbb{R}$ is called an almost periodic sequence if for every $\varepsilon > 0$,

$$P(\varepsilon, f) = \{\tau \in \mathbb{Z} : |f(n+\tau) - f(n)| < \varepsilon \text{ for all } n \in \mathbb{Z}\}$$

is a relatively dense set in $\mathbb{Z}$. Denote by $AP(\mathbb{Z}, \mathbb{R})$ the set of all such functions.

Denote by $PAP_0(\mathbb{Z}, \mathbb{R})$ the space of all bounded functions $f : \mathbb{Z} \to \mathbb{R}$ such that

$$\lim_{n\to\infty} \frac{1}{2n} \sum_{k=-n}^{n} |f(k)| = 0.$$

Definition 3. A function $f : \mathbb{Z} \to \mathbb{R}$ is called pseudo almost periodic sequence if it admits a decomposition $f = g + h$, where $g \in AP(\mathbb{Z}, \mathbb{R})$ and $h \in PAP_0(\mathbb{Z}, \mathbb{R})$. Denote by $PAP(\mathbb{Z}, \mathbb{R})$ the set of all such functions.

Lemma 1. Let $X \in \{AP(\mathbb{Z}, \mathbb{R}), PAP(\mathbb{Z}, \mathbb{R})\}$. Then the following hold true:

(a) $f \in X$ implies that f is bounded.

(b) $f, g \in X$ imply that $f + g, f \cdot g \in X$. Moreover, $f, g \in X$ implied that $f/g \in X$ provided that $\inf_{n\in\mathbb{Z}} |g(n)| > 0$.

(c) X is a Banach space equipped with the supremum norm;

(d) $f \in X$ implies that $f(\cdot + k) \in X$ for every $k \in \mathbb{Z}$;

(e) $f \in X$ implies that $F \circ f \in X$ for every continuous function $F : \mathbb{R} \to \mathbb{R}$.

Now, let us recall some basic results about the difference system

$$x(n+1) = A(n)x(n), \quad n \in \mathbb{Z}, \tag{1.3}$$

where for every $n \in \mathbb{Z}$, $x(n) \in \mathbb{R}^q$ and $A(n)$ is an invertible $q \times q$ matrix. Denote

$$\Phi(n, m) = \begin{cases} A(n-1)\cdots A(m), & n > m, \\ I, & n = m, \\ A^{-1}(n)\cdots A^{-1}(m), & n < m. \end{cases}$$

Definition 4. [20] We call that the linear difference system (1.3) has an exponential dichotomy on $\mathbb{Z}$ if there are positive constants K, λ and a family of projection $P(n)$ such that

$$P(n+1)A(n) = A(n)P(n), \quad n \in \mathbb{Z},$$

and

$$|\Phi(n,m)P(m)| \leq Ke^{-\lambda(n-m)},\ n \geq m, \quad |\Phi(n,m)(I-P(m))| \leq Ke^{-\lambda(m-n)},\ m > n.$$

Lemma 2. [20] If the linear difference system (1.3) has an exponential dichotomy on $\mathbb{Z}$ and $f : \mathbb{Z} \to \mathbb{R}$ is bounded, then the inhomogeneous system

$$x(n+1) = A(n)x(n) + f(n), \quad n \in \mathbb{Z},$$

has a unique bounded solution given by

$$x(n) = \sum_{m=-\infty}^{n-1} \Phi(n,m+1)P(m+1)f(m) - \sum_{m=n}^{+\infty} \Phi(n,m+1)(I-P(m+1))f(m), \quad n \in \mathbb{Z}.$$

The following two lemmas are due to [13].

Lemma 3. Let $\phi \in PAP(\mathbb{Z},\mathbb{R})$ and $\tau \in PAP(\mathbb{Z},\mathbb{Z})$. Then, $\phi(\cdot - \tau(\cdot)) \in PAP(\mathbb{Z},\mathbb{R})$.

Lemma 4. Let $\theta, f \in PAP(\mathbb{Z},\mathbb{R})$ with $\|\theta\| < 1$, and

$$F(n) = \sum_{m=-\infty}^{n-1} \left[\prod_{i=m+1}^{n-1} \theta(i) \right] f(m), \quad n \in \mathbb{Z},$$

where we denote $\prod_{i=n}^{n-1} \theta(i) = 1$ for simplicity. Then $F \in PAP(\mathbb{Z},\mathbb{R})$.

2 Main results

Throughout the rest of this paper, for every subset $\Omega \subset \mathbb{R}$, we denote by $PAP(\mathbb{Z},\Omega)$ the set of all functions $f \in PAP(\mathbb{Z},\mathbb{R})$ with $f(\mathbb{Z}) \subset \Omega$. We also use the notations $AP(\mathbb{Z},\Omega)$ and $PAP_0(\mathbb{Z},\Omega)$, which is similar to $PAP(\mathbb{Z},\Omega)$.

For the next existence theorem, we will use the following assumptions:

(A0) $\alpha \in PAP(\mathbb{Z},\mathbb{R})$ with $0 < \alpha^- \leq \alpha^+ < 1$, $b, \beta, \gamma \in PAP(\mathbb{Z},\mathbb{R}^+)$ with $b^-, \beta^-, \gamma^- > 0$ and $\tau \in PAP(\mathbb{Z},\mathbb{Z}^+)$.

(A1) There exist $\Gamma_1 \geq \Gamma_2 \geq 1$ such that

$$\frac{b^+\Gamma_2^{\gamma^-}+\beta^+\Gamma_1\ln\Gamma_1}{\alpha^-\Gamma_2^{\gamma^-}} \leq \Gamma_1, \quad \frac{b^-\Gamma_1^{\gamma^+}+\beta^-\Gamma_2\ln\Gamma_2}{\alpha^+\Gamma_1^{\gamma^+}} \geq \Gamma_2.$$

(A2) $\beta^+(\Gamma_1\Gamma_2^{\gamma^--1}+\Gamma_2^{\gamma^-}\ln\Gamma_1+\gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1) < \alpha^-\Gamma_2^{2\gamma^-}$.

Theorem 1. Let (A0)-(A2) hold. Then there exists a unique pseudo almost periodic solution of equation (1.2) in

$$\Psi^* = \{\varphi \in PAP(\mathbb{Z},\mathbb{R}),\ \Gamma_2 \leq \varphi(n) \leq \Gamma_1, \forall n \in \mathbb{Z}\}.$$

Proof. Fix $\phi \in \Psi^*$. Let us consider an auxiliary equation

$$\Delta x(n) = -\alpha(n)x(n) + b(n) + \beta(n)\ln\phi(n-\tau(n)) \cdot \frac{\phi(n)}{\phi^{\gamma(n)}(n-\tau(n))}, \tag{2.1}$$

i.e.,

$$x(n+1) = (1-\alpha(n))x(n) + b(n) + \beta(n)\ln\phi(n-\tau(n)) \cdot \frac{\phi(n)}{\phi^{\gamma(n)}(n-\tau(n))}.$$

Noting that $0 < \alpha^- \leq \alpha^+ < 1$, we know that the linear equation $x(n+1) = (1-\alpha(n))x(n)$ admits an exponential dichotomy on $\mathbb{Z}$ with $P(n) \equiv I$.

Noting that

$$\phi^{\gamma(n)}(n-\tau(n)) = e^{\ln\phi^{\gamma(n)}(n-\tau(n))} = e^{\gamma(n)\ln\phi(n-\tau(n))},$$

by (A0), Lemma 3 and (e) of Lemma 1, we conclude that $n \to \phi^{\gamma(n)}(n-\tau(n))$ is in $PAP(\mathbb{Z},\mathbb{R})$. Again by (A0), Lemma 3, and (b) of Lemma 1, in view of

$$\inf_{n\in\mathbb{Z}} \phi^{\gamma(n)}(n-\tau(n)) \geq \Gamma_2^{\gamma^-},$$

we deduce that

$$n \to b(n) + \beta(n)\ln\phi(n-\tau(n)) \cdot \frac{\phi(n)}{\phi^{\gamma(n)}(n-\tau(n))}$$

is in $PAP(\mathbb{Z},\mathbb{R})$.

Then, by Lemma 2, equation (2.1) has a unique bounded solution given by

$$x^{\phi}(n) = \sum_{k=-\infty}^{n-1}\left[\prod_{r=k+1}^{n-1}(1-\alpha(r))\right] \cdot \left[b(k) + \beta(k)\ln\phi(k-\tau(k)) \cdot \frac{\phi(k)}{\phi^{\gamma(k)}(k-\tau(k))}\right].$$

Define a mapping T on Ψ^* by

$$(T\phi)(n) = x^{\phi}(n),\ \ \phi \in \Psi^*,\ n \in \mathbb{Z}.$$

It follows from $\alpha \in PAP(\mathbb{Z},\mathbb{R})$, $\|1-\alpha\| \leq 1-\alpha^- < 1$, and Lemma 4 that $T(\Psi^*) \subset PAP(\mathbb{Z},\mathbb{R})$.

Next, let us show that $T(\Psi^*) \subset \Psi^*$. For every $\phi \in \Psi^*$ and $n \in \mathbb{Z}$, we have

$$\begin{aligned}(T\phi)(n) &\leq \sum_{k=-\infty}^{n-1}\prod_{r=k+1}^{n-1}(1-\alpha^-)(b^+ + \beta^+ \ln\Gamma_1 \cdot \frac{\Gamma_1}{\Gamma_2^{\gamma^-}})\\ &\leq \sum_{k=-\infty}^{n-1}(1-\alpha^-)^{n-k-1}(b^+ + \beta^+ \ln\Gamma_1 \cdot \frac{\Gamma_1}{\Gamma_2^{\gamma^-}})\\ &= \frac{b^+ + \beta^+ \ln\Gamma_1 \cdot \frac{\Gamma_1}{\Gamma_2^{\gamma^-}}}{\alpha^-} \leq \Gamma_1.\end{aligned}$$

Also, for every $\phi \in \Psi^*$ and $n \in \mathbb{Z}$, we have

$$\begin{aligned}(T\phi)(n) &\geq \sum_{k=-\infty}^{n-1}\prod_{r=k+1}^{n-1}(1-\alpha^+)(b^- + \beta^- \ln\Gamma_2 \cdot \frac{\Gamma_2}{\Gamma_1^{\gamma^+}})\\ &= \sum_{k=-\infty}^{n-1}(1-\alpha^+)^{n-k-1}(b^- + \beta^- \ln\Gamma_2 \cdot \frac{\Gamma_2}{\Gamma_1^{\gamma^+}})\\ &= \frac{b^-\Gamma_1^{\gamma^+} + \beta^-\Gamma_2 \ln\Gamma_2}{\alpha^+\Gamma_1^{\gamma^+}} \geq \Gamma_2.\end{aligned}$$

This tells that T is a self-mapping from Ψ^* to Ψ^*.

For all $\varphi,\ \psi \in \Psi^*$, there holds

$$\begin{aligned}&\|T(\varphi) - T(\psi)\|\\ =& \sup_{n\in\mathbb{Z}}|(T\varphi)(n) - (T\psi)(n)|\\ =& \sup_{n\in\mathbb{Z}}|\sum_{k=-\infty}^{n-1}\prod_{r=k+1}^{n-1}[1-\alpha(r)]\beta(k)[\ln\varphi(k-\tau(k)) \cdot \frac{\varphi(k)}{\varphi^{\gamma(k)}(k-\tau(k))}\\ &-\ln\psi(k-\tau(k)) \cdot \frac{\psi(k)}{\psi^{\gamma(k)}(k-\tau(k))}]|\\ =& \sup_{n\in\mathbb{Z}}|\sum_{k=-\infty}^{n-1}\prod_{r=k+1}^{n-1}[1-\alpha(r)]\beta(k)\{\frac{\varphi(k)}{\varphi^{\gamma(k)}(k-\tau(k))}[\ln\varphi(k-\tau(k)) - \ln\psi(k-\tau(k))]\\ &+\frac{\ln\psi(k-\tau(k))}{\psi^{\gamma(k)}(k-\tau(k))}[\varphi(k)-\psi(k)]\\ &-\frac{\ln\psi(k-\tau(k))\cdot\varphi(k)}{\varphi^{\gamma(k)}(k-\tau(k))\cdot\psi^{\gamma(k)}(k-\tau(k))}[\varphi^{\gamma(k)}(k-\tau(k)) - \psi^{\gamma(k)}(k-\tau(k))]\}|\\ \leq& \sup_{n\in\mathbb{Z}}\sum_{k=-\infty}^{n-1}\prod_{r=k+1}^{n-1}(1-\alpha^-)\beta^+\{\frac{\varphi(k)}{\varphi^{\gamma(k)}(k-\tau(k))}\cdot\frac{1}{\Gamma_2}\|\varphi-\psi\| + \frac{\ln\psi(k-\tau(k))}{\psi^{\gamma(k)}(k-\tau(k))}\cdot\|\varphi-\psi\|\\ &+\frac{\ln\psi(k-\tau(k))\cdot\varphi(k)}{\varphi^{\gamma(k)}(k-\tau(k))\cdot\psi^{\gamma(k)}(k-\tau(k))}\cdot\gamma^+\Gamma_1^{\gamma^+-1}\|\varphi-\psi\|\end{aligned}$$

$$\le \sup_{n\in\mathbb{Z}} \sum_{k=-\infty}^{n-1} \prod_{r=k+1}^{n-1} (1-\alpha^-)\beta^+ \left(\frac{\Gamma_1}{\Gamma_2^{\gamma^-}} \cdot \frac{1}{\Gamma_2} + \frac{\ln\Gamma_1}{\Gamma_2^{\gamma^-}} + \frac{\ln\Gamma_1\cdot\Gamma_1}{\Gamma_2^{\gamma^-}\cdot\Gamma_2^{\gamma^-}} \cdot \gamma^+\Gamma_1^{\gamma^+-1} \right) \|\varphi-\psi\|$$
$$= \frac{\beta^+(\Gamma_1\Gamma_2^{\gamma^--1} + \Gamma_2^{\gamma^-}\ln\Gamma_1 + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1)}{\alpha^-\Gamma_2^{2\gamma^-}} \cdot \|\varphi-\psi\|.$$

By $(A2)$, T is a contraction. Therefore, T has a unique fixed point in Ψ^*, which means that equation (1.2) has a unique pseudo almost periodic solution in Ψ^*. This completes the proof.

Next, let us discuss the exponential stability of pseudo almost periodic solution of equation (1.2).

Theorem 2. Suppose that (A0)-(A2) are satisfied, x_* be the unique pseudo almost periodic solution of equation (1.2) in Ψ^*, and x be an arbitrary solution of equation (1.2) satisfying that there exists $N\in\mathbb{N}$ such that $\Gamma_2 \le x(n) \le \Gamma_1$ for all $n \ge N$. Moreover, there holds

$$\beta^+(\Gamma_1\Gamma_2^{\gamma^--1} + \Gamma_2^{\gamma^-}\ln\Gamma_1 + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1) < \frac{1-\alpha^+}{1-\alpha^-} \cdot \alpha^-\Gamma_2^{2\gamma^-}. \tag{2.2}$$

Then, there is a constant $\lambda > 0$ such that

$$|x(n) - x_*(n)| \le Me^{-\lambda n}, \quad n \ge N,$$

where $M = e^{\lambda N} \cdot \max_{N-\tau^+\le n\le N} |x(n) - x_*(n)|$.

Proof. Let $y(n) = x(n) - x_*(n)$, $n \ge N$. Then

$$\triangle y(n) = -\alpha(n)y(n) + \beta(n) \left[\frac{x(n)\ln x(n-\tau(n))}{x^{\gamma(n)}(n-\tau(n))} - \frac{x_*(n)\ln x_*(n-\tau(n))}{x_*^{\gamma(n)}(n-\tau(n))} \right],$$

i.e.,

$$\triangle y(n) = -\frac{\alpha(n)}{1-\alpha(n)} y(n+1) + \frac{\beta(n)}{1-\alpha(n)} \left[\frac{x(n)\ln x(n-\tau(n))}{x^{\gamma(n)}(n-\tau(n))} - \frac{x_*(n)\ln x_*(n-\tau(n))}{x_*^{\gamma(n)}(n-\tau(n))} \right]. \tag{2.3}$$

By (2.2), we can choose a sufficiently small $\lambda > 0$ such that

$$\frac{\beta^+\left(\Gamma_1\Gamma_2^{\gamma^--1}\cdot e^{\lambda(\tau^++1)} + \Gamma_2^{\gamma^-}\ln\Gamma_1\cdot e^{\lambda} + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1\cdot e^{\lambda(\tau^++1)}\right)}{(1-\alpha^+)\Gamma_2^{2\gamma^-}} < \frac{\alpha^-}{1-\alpha^-} + 1 - e^{\lambda}. \tag{2.4}$$

Consider the discrete Lyapunov functional

$$V(n) = |y(n)|e^{\lambda n}, \quad n \ge N.$$

It is easy to see that $V(n) \le M$ for all $n \in [N-\tau^+, N]$. We claim that $V(n) \le M$ for all $n > N$. In fact, if this is not true,

$$\{n > N : V(n) > M\} \neq \emptyset.$$

Set

$$n_0 = \min\{n > N : V(n) > M\} - 1.$$

Then $n_0 \ge N$ and

$$V(n_0+1) > M, \quad V(n) \le M,\ n \in [N-\tau^+, n_0].$$

In addition, without loss of generality, we can assume that $\Gamma_2 \le x(n) \le \Gamma_1$ for all $n \ge N - \tau^+$. Then, by (2.3) and (2.4), we have

$$\begin{aligned}
&\Delta V(n_0)\\
&= \Delta(|y(n_0)|e^{\lambda n_0})\\
&= \Delta|y(n_0)|e^{\lambda(n_0+1)} + |y(n_0)|\Delta e^{\lambda n_0}\\
&\le \frac{-\alpha(n_0)}{1-\alpha(n_0)}|y(n_0+1)|e^{\lambda(n_0+1)} + |y(n_0)|(e^{\lambda(n_0+1)} - e^{\lambda n_0})\\
&\quad + \frac{\beta(n_0)}{1-\alpha(n_0)}\left|\frac{x(n_0)\ln x(n_0-\tau(n_0))}{x^{\gamma(n_0)}(n_0-\tau(n_0))} - \frac{x_*(n_0)\ln x_*(n_0-\tau(n_0))}{x_*^{\gamma(n_0)}(n_0-\tau(n_0))}\right| e^{\lambda(n_0+1)}\\
&\le -\frac{\alpha^-}{1-\alpha^-}M + M(e^\lambda - 1) + \frac{\beta^+}{1-\alpha^+}\left|\frac{x(n_0)\ln x(n_0-\tau(n_0))}{x^{\gamma(n_0)}(n_0-\tau(n_0))} - \frac{x_*(n_0)\ln x_*(n_0-\tau(n_0))}{x_*^{\gamma(n_0)}(n_0-\tau(n_0))}\right| e^{\lambda(n_0+1)}\\
&\le M(e^\lambda - 1 - \frac{\alpha^-}{1-\alpha^-})\\
&\quad + \frac{\beta^+\left(\Gamma_1\Gamma_2^{\gamma^- -1}\cdot|y(n_0-\tau(n_0))| + \Gamma_2^{\gamma^-}\ln\Gamma_1\cdot|y(n_0)| + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1\cdot|y(n_0-\tau(n_0))|\right)}{(1-\alpha^+)\Gamma_2^{2\gamma^-}} e^{\lambda(n_0+1)}\\
&\le M(e^\lambda - 1 - \frac{\alpha^-}{1-\alpha^-}) + \frac{M\beta^+\left(\Gamma_1\Gamma_2^{\gamma^- -1}\cdot e^{\lambda(\tau^++1)} + \Gamma_2^{\gamma^-}\ln\Gamma_1\cdot e^{\lambda} + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1\cdot e^{\lambda(\tau^++1)}\right)}{(1-\alpha^+)\Gamma_2^{2\gamma^-}}\\
&= M\left[e^\lambda - 1 - \frac{\alpha^-}{1-\alpha^-} + \frac{\beta^+\left(\Gamma_1\Gamma_2^{\gamma^- -1}\cdot e^{\lambda(\tau^++1)} + \Gamma_2^{\gamma^-}\ln\Gamma_1\cdot e^{\lambda} + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1\cdot e^{\lambda(\tau^++1)}\right)}{(1-\alpha^+)\Gamma_2^{2\gamma^-}}\right]\\
&< 0,
\end{aligned}$$

which is a contradiction. So $V(n) \le M$ for all $n \ge N$, i.e.,

$$|x(n) - x^*(n)| \le Me^{-\lambda n}, \quad n \ge N.$$

At last, we give an example to show that all the assumptions in the above two theorems can be satisfied.

Example 1. Let $\alpha,\beta,\gamma,b \in PAP(\mathbb{Z},\mathbb{R})$ and $\tau \in PAP(\mathbb{Z},\mathbb{Z}^+)$ be such that

$$\alpha^- = 0.3,\ \alpha^+ = 0.4, \quad \beta^- = 0.1,\ \beta^+ = 0.2,$$

and

$$\gamma^+ = \gamma^- = 1, \quad b^- = 40,\ b^+ = 41.$$

So (A0) holds. Choosing $\Gamma_2 = e^4 < \Gamma_1 = 4e^4$, we have

$$\frac{b^+\Gamma_2^{\gamma^-} + \beta^+\Gamma_1 \ln\Gamma_1}{\alpha^-\Gamma_2^{\gamma^-}} \approx 151.030 < \Gamma_1 = 4e^4 \approx 218.393,$$

and

$$\frac{b^-\Gamma_1^{\gamma^+} + \beta^-\Gamma_2 \ln\Gamma_2}{\alpha^+\Gamma_1^{\gamma^+}} = 100.25 > \Gamma_2 = e^4 \approx 54.598$$

Thus, (A1) holds. Moreover,

$$\frac{\beta^+(\Gamma_1\Gamma_2^{\gamma^- -1} + \Gamma_2^{\gamma^-}\ln\Gamma_1 + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1)}{\alpha^-\Gamma_2^{2\gamma^-}} \approx 0.378 < 1,$$

which means that (A2) holds, and

$$\frac{\beta^+(\Gamma_1\Gamma_2^{\gamma^- -1} + \Gamma_2^{\gamma^-}\ln\Gamma_1 + \gamma^+\Gamma_1^{\gamma^+}\ln\Gamma_1)}{\alpha^-\Gamma_2^{2\gamma^-}} < \frac{1-\alpha^+}{1-\alpha^-} = \frac{6}{7},$$

i.e., (2.2) holds. Thus, all the assumptions of Theorem 1 and Theorem 2 are satisfied.

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